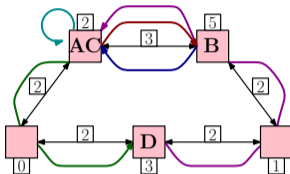


Approximate Graph Embeddings in the Cloud



Highlights of Algorithms 2018

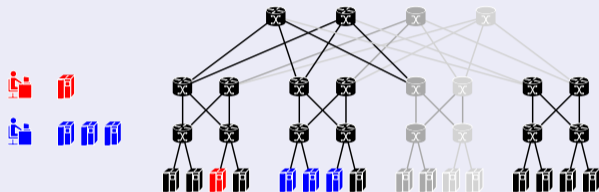
Matthias Rost

Technische Universität Berlin, Internet Network Architectures

Stefan Schmid

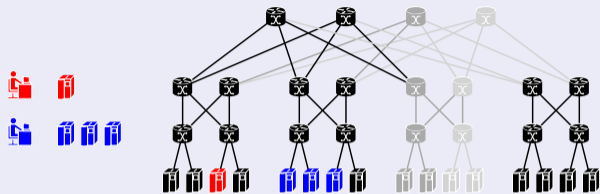
Universität Wien, Communication Technologies

Cloud Providers Offer Data Center Resources



Customers Cloud Data Center (Amazon, Google, ...)

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'Classic' Cloud Computing

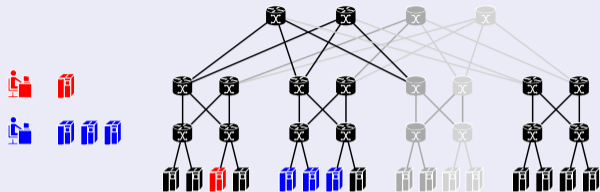
- ▶ Customer specifies number and 'size' of Virtual Machines



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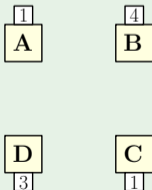
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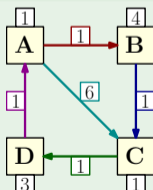
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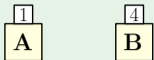
Goal: Virtual Networks (since \approx 2006)

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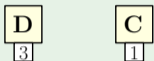


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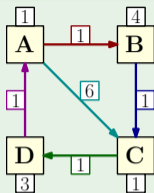
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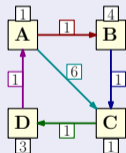


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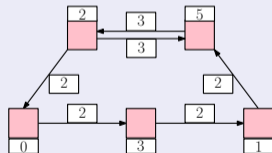
The Virtual Network Embedding Problem (VNEP)

- ▶ Map virtual nodes to substrate nodes
- ▶ Map virtual edges to paths in the substrate
- ▶ Respecting capacities & mapping restrictions

Virtual Network

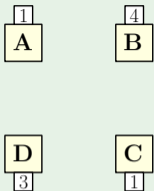


Substrate (Physical Network)



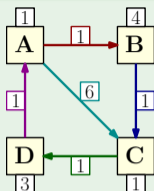
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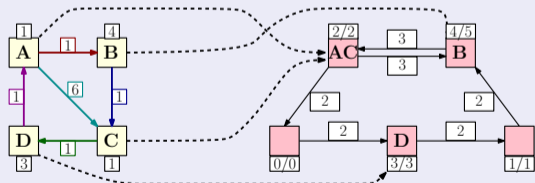
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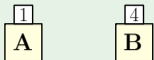
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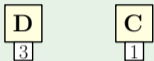


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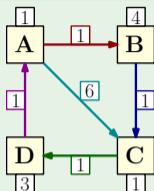


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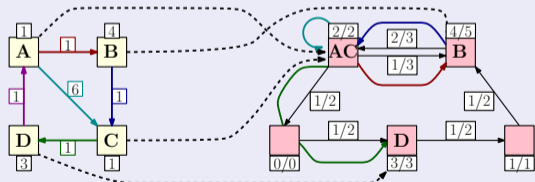
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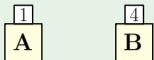
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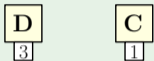


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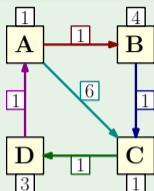
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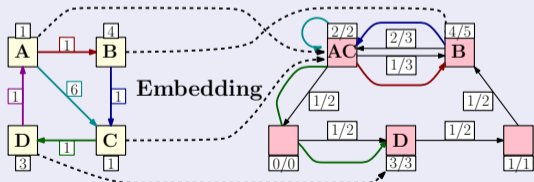


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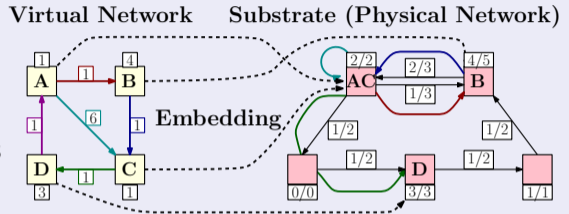
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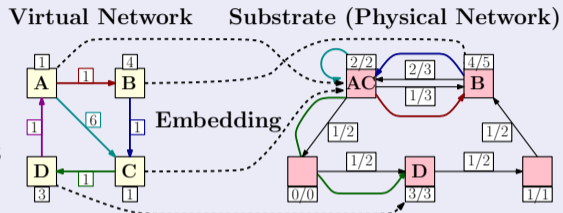


Related Work

- ▶ VNEP (and related problems) studied intensively in the networking community: > 100 papers.
- ▶ VNEP is related to classical problems as, e.g., subgraph isomorphism, but different ...
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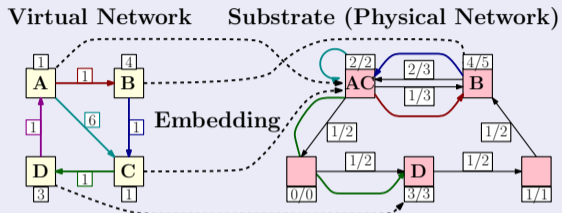


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Focus: Offline Variant

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Objectives Maximize profit (admission control) or minimize 'cost' *s.t.* *capacity constraints*.

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Approach: Randomized Rounding à la Raghavan & Thompson

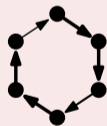
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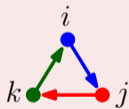
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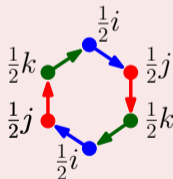
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Substrate



Request



Classic LP Solution

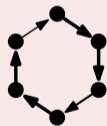
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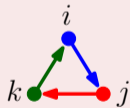
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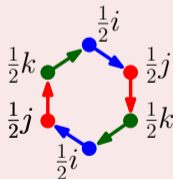
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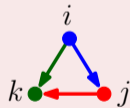
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Extraction Order

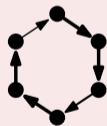
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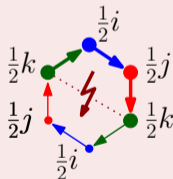
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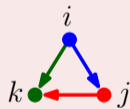
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Request



Classic LP Solution



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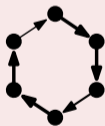
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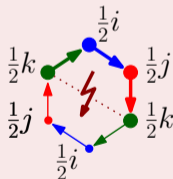
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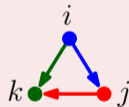
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