Approximating Geometric Knapsack via L-Packings

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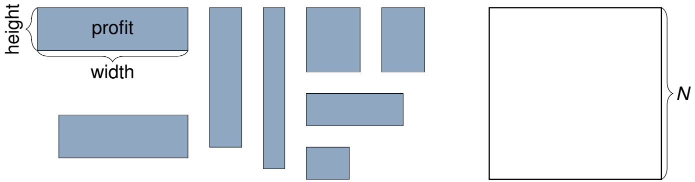
joint work with Fabrizio Grandoni, Sandy Heydrich, Salvatore Ingala, Arindam Khan and Andreas Wiese (FOCS 2017)

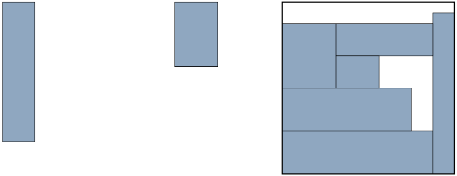
Two-dimensional Geometric Knapsack

- Items, represented by axis-parallel rectangles, characterized by a height, width and profit;
- A rectangular knapsack of dimension $N \times N$.



Two-dimensional Geometric Knapsack

- Items, represented by axis-parallel rectangles, characterized by a height, width and profit;
- A rectangular knapsack of dimension $N \times N$.
- Find a non-overlapping packing of a subset of the items while maximizing the profit.



- Applications: Cutting stock, Advertising placement, Scheduling on consecutive resources, ...

Variants and known results

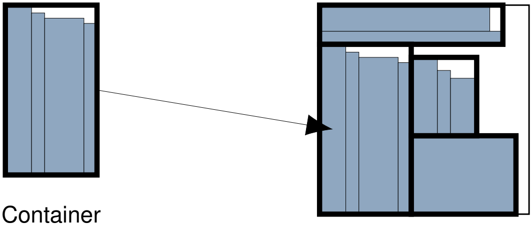
- Some variants of the problem have been studied:
  - Cardinality case: All the profits are equal to 1.
  - Rotations allowed: Items can be rotated by 90 degrees.
- It is strongly NP-hard to check if a given set of squares can be packed into a given knapsack. [Leung et al., 1990]
- For all the mentioned variants, the best approximation algorithms known with polynomial running time have approximation ratio $2 + \varepsilon$. [Jansen & Zhang, 2004]

Our Results

The barrier of 2 can be broken for all the mentioned variants.

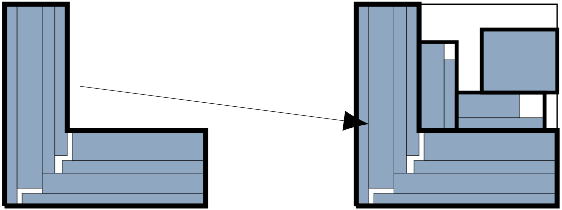
Previous approach: Containers

- A (vertical) container is a box where items inside are packed one next to each other (each vertical line intersects at most one item inside).
- Solutions decomposed into a constant number of containers can be found efficiently via Dynamic Programming losing negligible profit.



Our approach: Containers + L-packing

- An L-packing is a packing of horizontal and vertical items into an L-shaped region.
- The new structure we propose: An L-packing plus containers in the remaining area.



L-Packing

Theorem

There is a $(1+\varepsilon)$-approximation for L-packings.

Improved results

Using this structural result plus additional techniques, we can get:

- A $\left(\frac{17}{9}+\varepsilon\right)$-approximation for the weighted case without rotations;
- A $\left(\frac{558}{325}+\varepsilon\right)$-approximation for the cardinality case without rotations;
- A $\left(\frac{3}{2}+\varepsilon\right)$-approximation for the weighted case with rotations; and
- A $\left(\frac{4}{3}+\varepsilon\right)$-approximation for the cardinality case with rotations.

Thank you