

Dynamics of Distributed Updating in Fisher Markets

(To Appear in ACM Conference on Economics and Computation (EC) 2018)

(Generalized / Damped
Proportional Response Dynamics)

Yun Kuen (Marco) CHEUNG

(Max Planck Institute for Informatics, Saarland Informatics Campus)

Richard COLE (Courant Institute, NYU)

Yixin TAO (Courant Institute, NYU)

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Take Home Message

**Markets Dynamics to
Reach Equilibrium**



**Optimization Iterative
Processes:**

Gradient/Mirror Descent



Take Home Message — Markets

JUSTIFY Market Equilibrium (ME)

- ME (and Nash Equilibrium) definition is static — provide no insight on how it can be arrived.
- For special markets, many P-time algorithms. (Ellipsoid Method, Interior Point Method, Flow-Based Combinatorial Algorithm)
- But they cannot be run in real market environment. (Distributed vs. Centralized)



Take Home Message — Markets

JUSTIFY Market Equilibrium (ME)

We need natural algorithm
which can be run
in the highly distributed
market environment.



Take Home Message — Markets

Natural Algorithms / Dynamics:

Tâtonnement (1874): Price Updates (by sellers)

Full Range of Fisher CES Markets

Equivalent to Gradient Descent

(C., Cole, Devanur, STOC 2013)

Proportional Response Dynamics (2007):

(motivated by peer-to-peer network sharing)

Spending Updates (by buyers)

Substitute Range of Fisher CES Markets

(Wu, Zhang STOC 2007; Zhang ICALP 2009)



Take Home Message — Markets

Proportional Response Dynamics (PRD):

Spending Updates (by buyers)

Substitute Range of Fisher CES Markets

OPEN: What about Complementary / Mixed ranges?

By establishing

equivalence to Mirror Descent,

we derive new form of PRD

which converges to ME in the

complementary / mixed ranges.



Take Home Message — Markets

For Complementary Range:

$$b_{ij}(t+1) \leftarrow \frac{a_{ij} \cdot (\text{total spending on good } j \text{ at time } t)^{\rho/(\rho-1)}}{\sum_k a_{ik} \cdot (\text{total spending on good } k \text{ at time } t)^{\rho/(\rho-1)}}$$

Recall that for Substitute Range:

$$b_{ij}(t+1) \leftarrow \frac{a_{ij} \cdot (\text{quantity of good } j \text{ obtained at time } t)^\rho}{\sum_k a_{ik} \cdot (\text{quantity of good } k \text{ obtained at time } t)^\rho}$$



Take Home Message — Optimization

Proportional Response Dynamics (PRD)

≡ Mirror Descent (MD)

PRD Converges to ME

≡ MD Converges to Minimum / Saddle Point

TECHNICAL HIGHLIGHTS:

We present new MD analyses to deal with
new and broad classes of functions.

(Bregman Convex or Convex-Concave Functions)



Take Home Message — Optimization

Convex-Concave Function:

$f(\mathbf{x}, \mathbf{y})$, convex in \mathbf{x} , concave in \mathbf{y}

$(\sigma_X, \sigma_Y, L_X, L_Y)$ -Strongly Bregman

Convex-Concave Function:

$$\begin{aligned} & -L_Y \cdot d_h(\mathbf{y}', \mathbf{y}) + \sigma_X \cdot d_g(\mathbf{x}', \mathbf{x}) \\ & \leq f(\mathbf{x}', \mathbf{y}') - \text{linearization at } f(\mathbf{x}, \mathbf{y}) \\ & \leq -\sigma_Y \cdot d_h(\mathbf{y}', \mathbf{y}) + L_X \cdot d_g(\mathbf{x}', \mathbf{x}). \end{aligned}$$

d_g, d_h are ANY Bregman divergences.



Take Home Message — Optimization

$(\sigma_X, \sigma_Y, L_X, L_Y)$ -Strongly Bregman Convex-Concave Function:

$$\begin{aligned} & -L_Y \cdot d_h(\mathbf{y}', \mathbf{y}) + \sigma_X \cdot d_g(\mathbf{x}', \mathbf{x}) \\ & \leq f(\mathbf{x}', \mathbf{y}') - \text{linearization at } f(\mathbf{x}, \mathbf{y}) \\ & \leq -\sigma_Y \cdot d_h(\mathbf{y}', \mathbf{y}) + L_X \cdot d_g(\mathbf{x}', \mathbf{x}). \end{aligned}$$

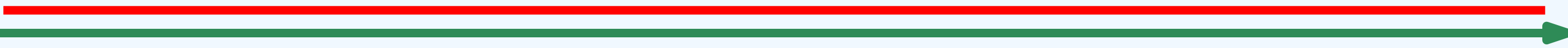
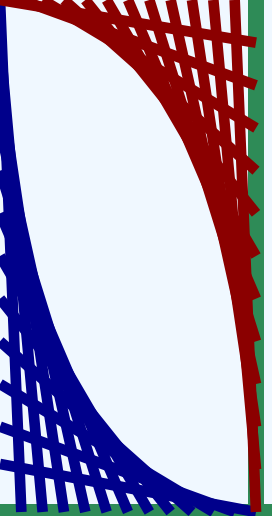
Update Rule:

$$\mathbf{x}^{t+1} \leftarrow \arg \min_{\mathbf{x}} \left\{ \langle \nabla_{\mathbf{x}} f(\mathbf{x}^t, \mathbf{y}^t), \mathbf{x} - \mathbf{x}^t \rangle + 2L_X \cdot d_g(\mathbf{x}, \mathbf{x}^t) \right\}$$

$$\mathbf{y}^{t+1} \leftarrow \arg \min_{\mathbf{y}} \left\{ -\langle \nabla_{\mathbf{y}} f(\mathbf{x}^t, \mathbf{y}^t), \mathbf{y} - \mathbf{y}^t \rangle + 2L_Y \cdot d_h(\mathbf{y}, \mathbf{y}^t) \right\}$$

THEOREM: (by a really clean analysis)

Pointwise Convergence to Saddle Point; Linear Convergence Rate.



**MORE DETAILS
AT POSTER**

19th Max Planck Advanced Course on the Foundations of Computer Science

13 - 17 August 2018, Saarbrücken, Germany



Fine-Grained Complexity and Algorithms



Ramamohan Paturi

UC San Diego

Foundations of Fine-grained Complexity



Amir Abboud

IBM Almaden

Hardness in P



Danupon Nanongkai

KTH

Dynamic graphs: algorithms, conditional lower bounds, and complexity classes

