Universal Framework for Wireless Scheduling

or

Hypergraph Sketches

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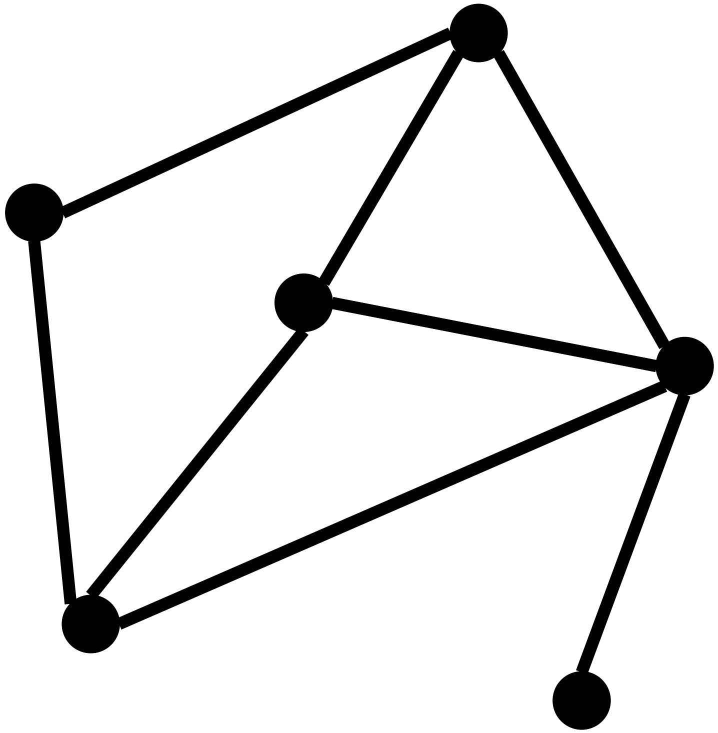
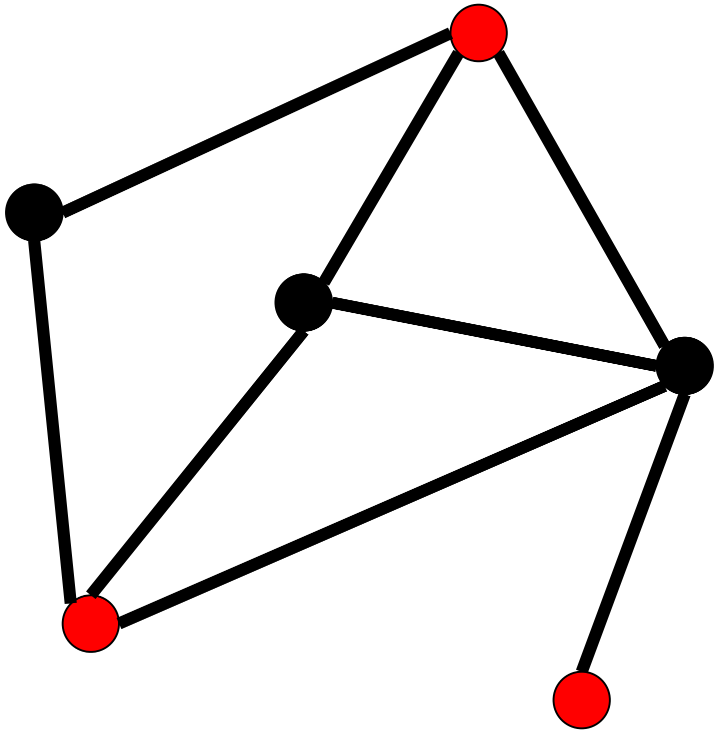
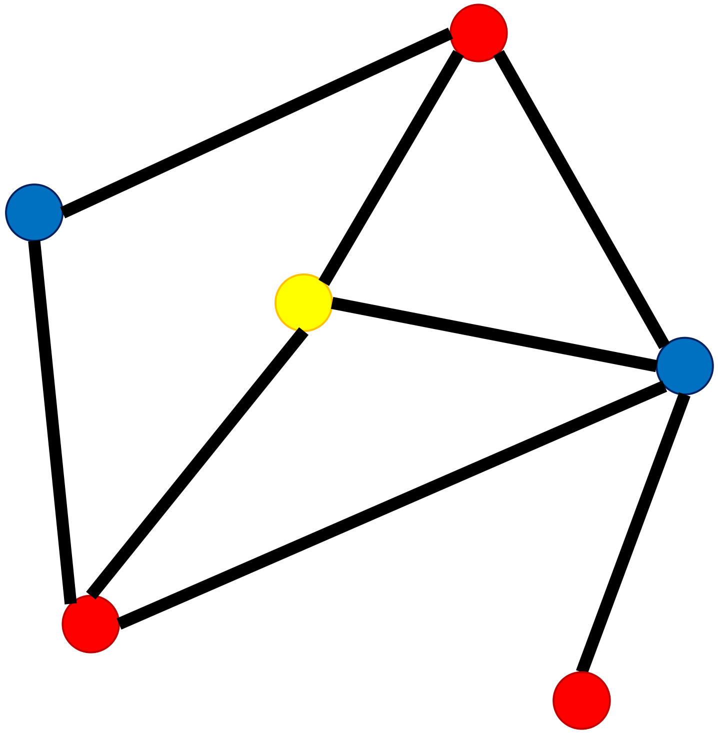
Graph Coloring : Dealing with Conflicts



Graph Coloring : Dealing with Conflicts

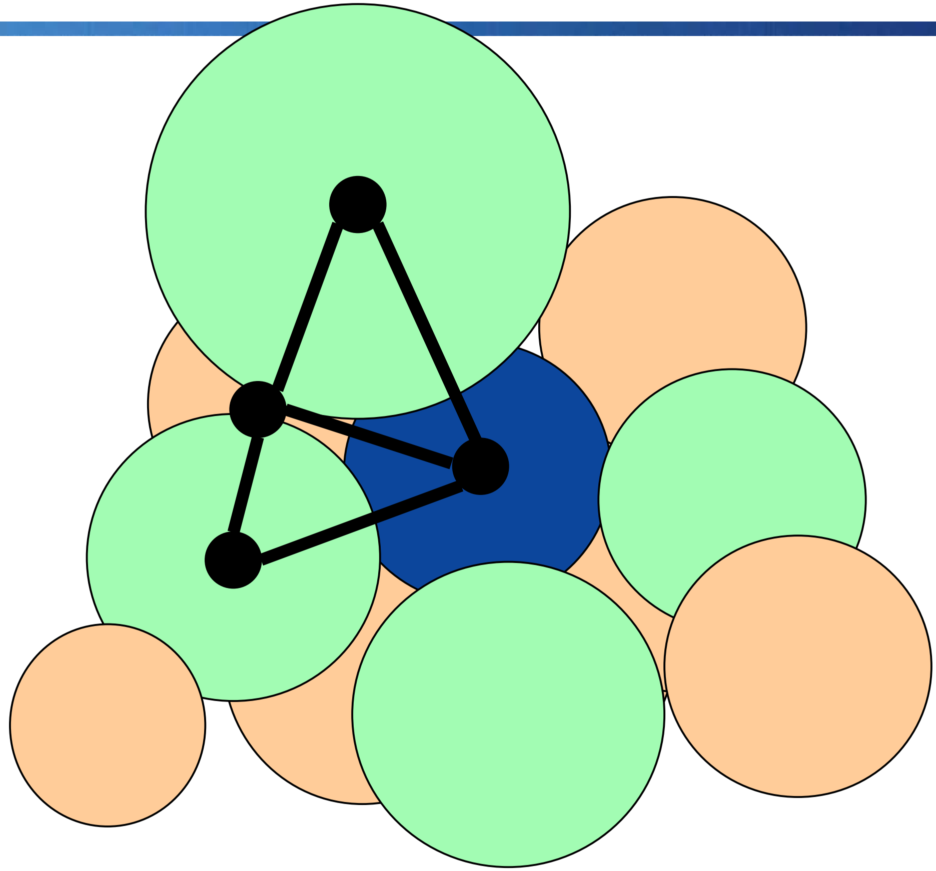


Graph Coloring : Dealing with Conflicts



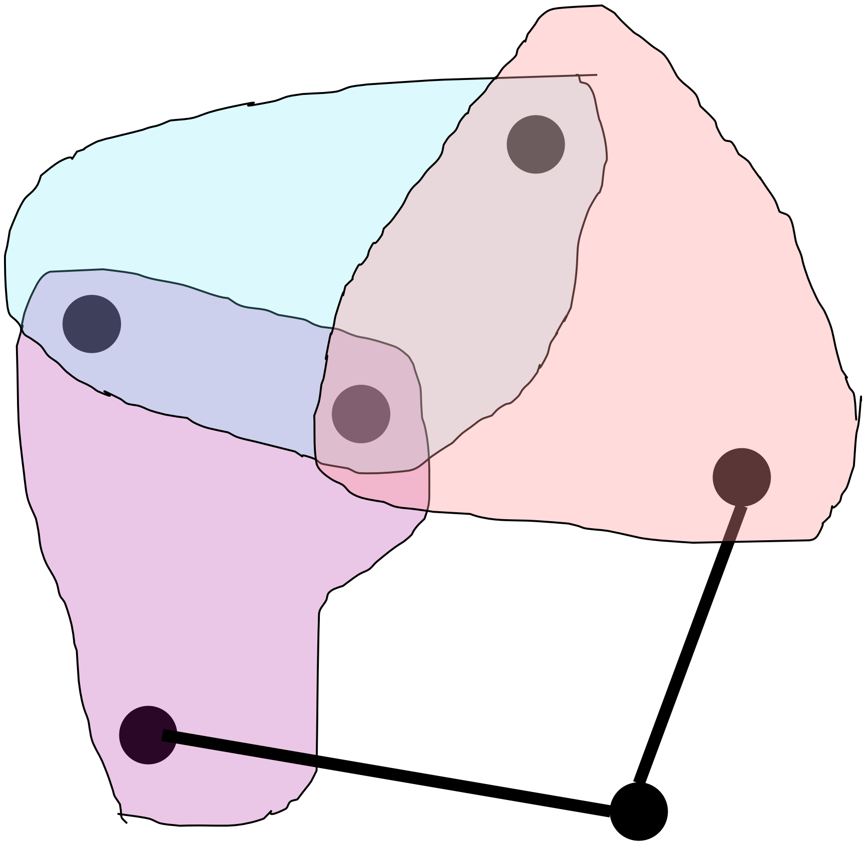
- But Graph Coloring is computationally hard

Disc Graphs & k-simplicial graphs



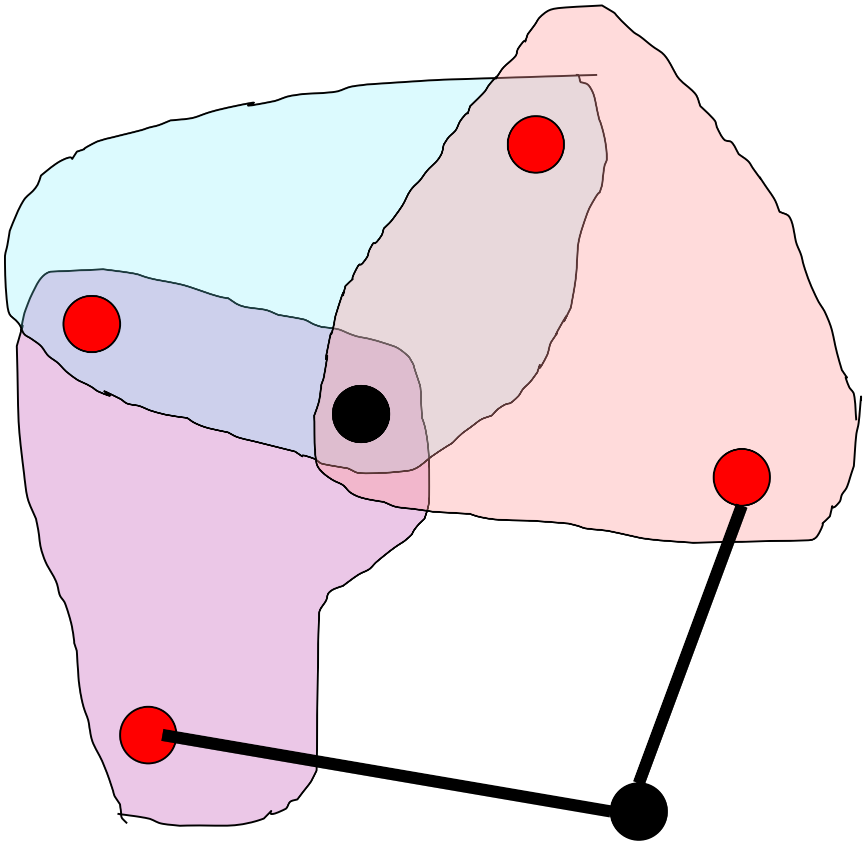
- Disk graphs are 6-simplicial: The neighborhood of the *smallest* disk can be covered with 6 cliques

Hypergraphs



- Hypergraphs capture more general constraints
  - „At most two out of these three can be active simultaneously“

Hypergraphs



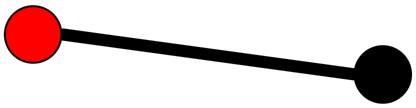
- Hypergraphs capture more general constraints
  - „At most two out of these three can be active simultaneously“

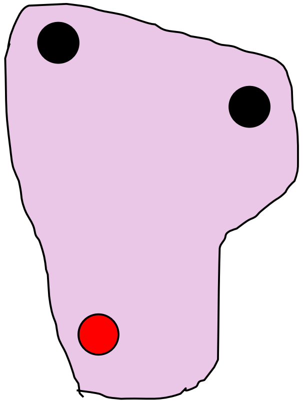
Hypergraphs: More general conflicts/constraints

So, why don‘t we use hypergraphs all the time?

- Because they are not very amenable
  - Less structure – less that can be done
  - Problems often become much harder
  - Limited theory that can be leveraged
  - More complex to analyze and conceptualize

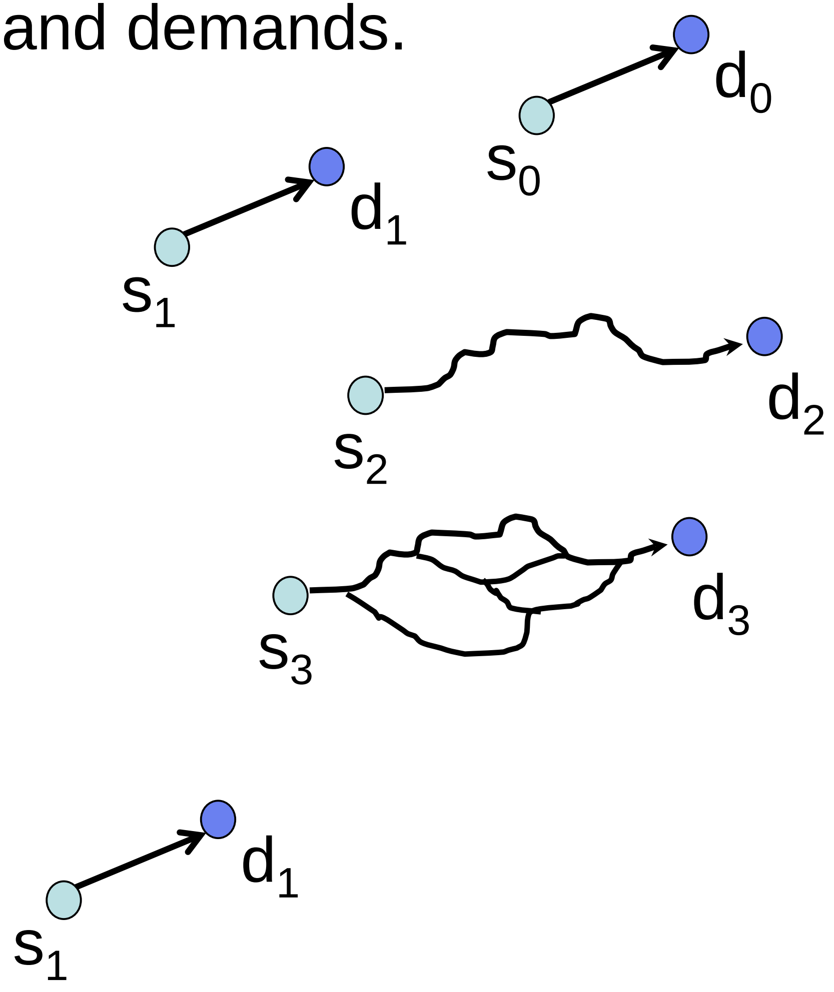
- An edge forces OPT to use different colors.
  Not so for a hyperedge.





Utilizing wireless networks

- *Given*: Sources, destinations, and demands.
- *Optimize*: „Throughput“
- *Using*:
  - Time multiplexing (= scheduling)
  - Channels (frequencies, codes)
  - Power control
  - Space diversity
  - Routing
  - Bit-rate control

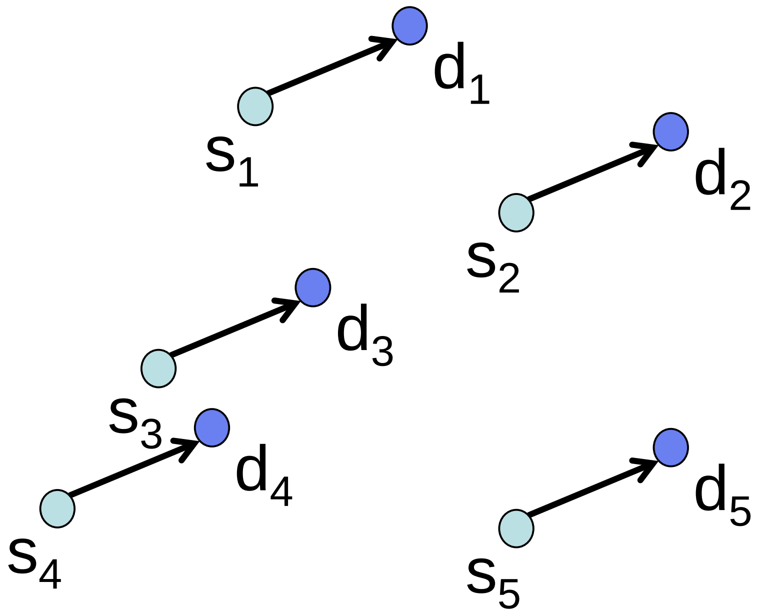


Core subproblem: Link Scheduling

- *(Shortest) Link Scheduling problem*

- *Given*: Links (sources, destinations)

- *Using*:
  - ~~Channels (frequencies, codes)~~
  - Power control
  - (TDMA) scheduling
  - Space diversity
  - ~~Routing~~
  - ~~Rate adjustment~~



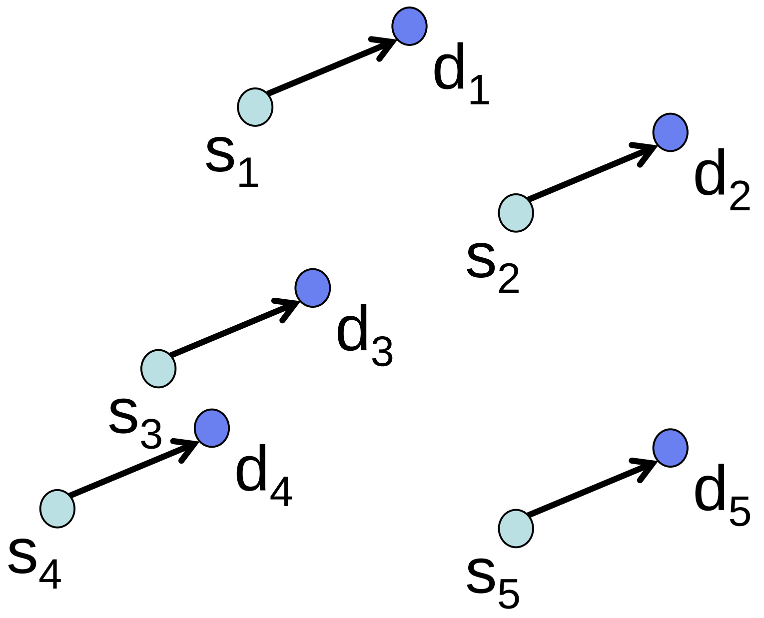
- *Minimize the number of slots used*

Related problem: MIS

- *MIS = „Max Independent Set of Links“,*

- *Given*: Links (sources, destinations)

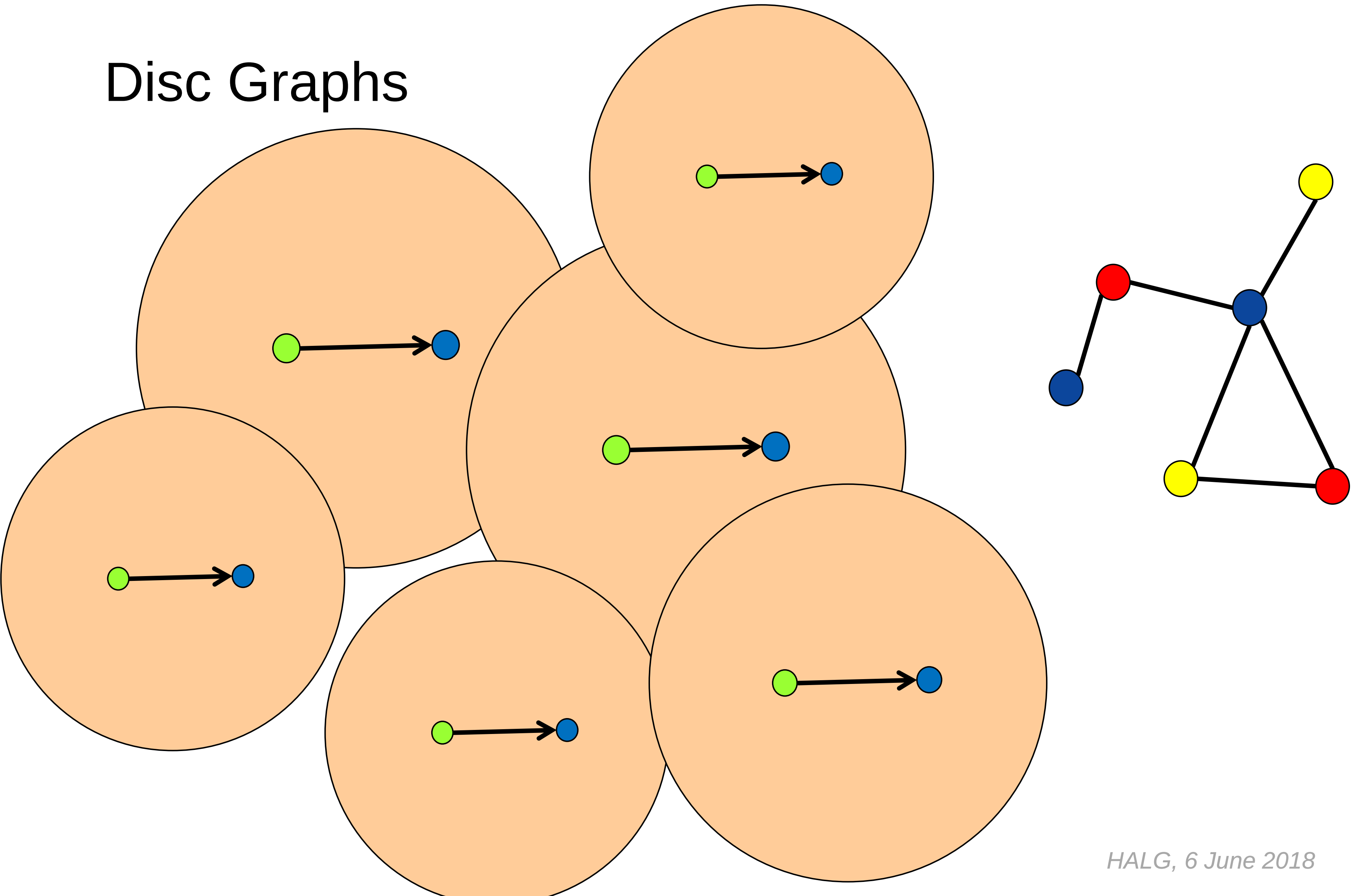
- *Using*:
  - ~~Channels (frequencies, codes)~~
  - Power control
  - ~~(TDMA) scheduling~~
  - Space diversity
  - ~~Routing~~
  - ~~Rate adjustment~~



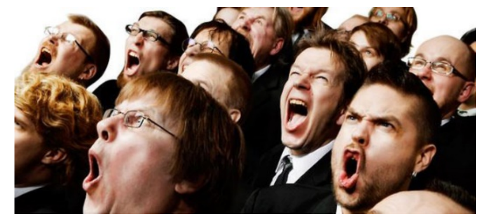
- *Maximize the number of links in a single slot*

Interference model : Disk Graphs

Disc Graphs



Interference: The challenging part of wireless



<u>What matters:</u>

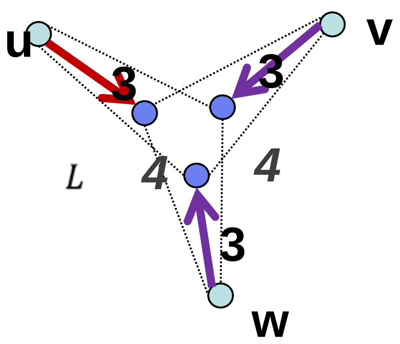
Is the received signal strength sufficiently large compared with the interference?

„Physical“ or SINR model

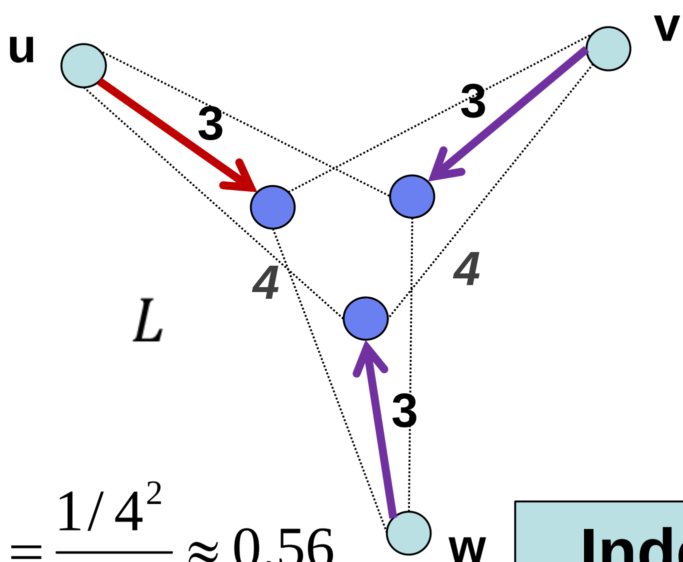
1. Additivity: Interference is *additive*

2. Threshold: Transmission succeeds if interference is below the „interference budget“

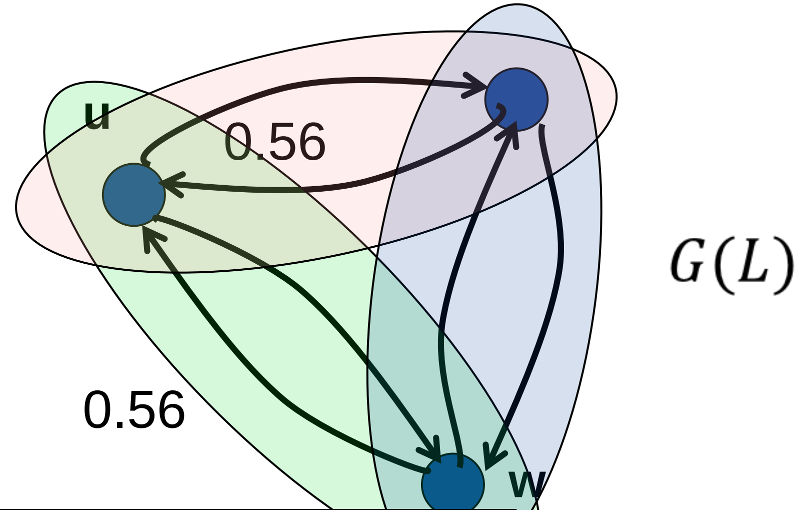
3. Distance based: Signal strength decreases polynomially with distance



Fractional conflict graphs



$$a_u(w) = \frac{1/4^2}{1/3^2} \approx 0.56$$

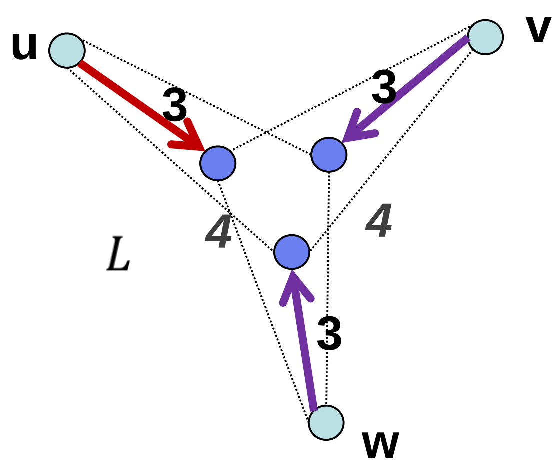


„Independent set“ in the conflict hypergraph

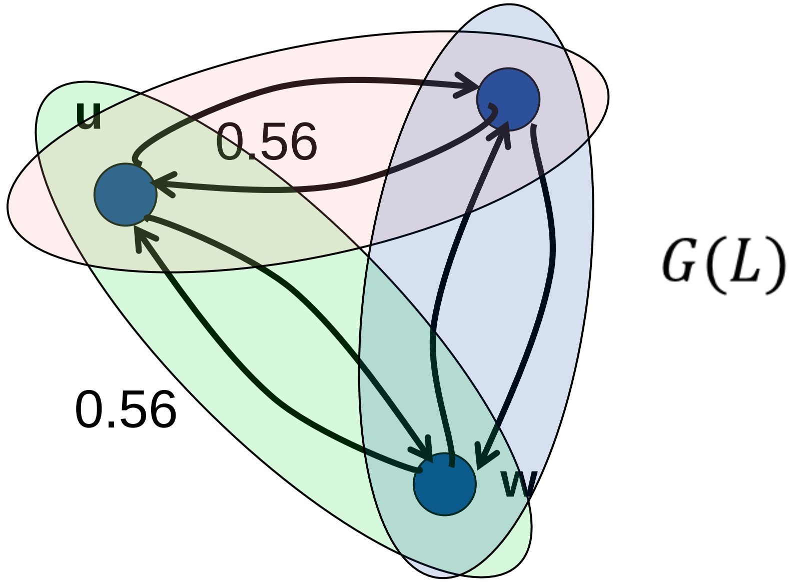
Given set L of links, aph $G(L)$.
Weight of edge ij = Re e interference of link i on link j

A set S is ***feasible*** iff
the weighted in-degree of every link within $G(S)$ is $< 1/\beta$

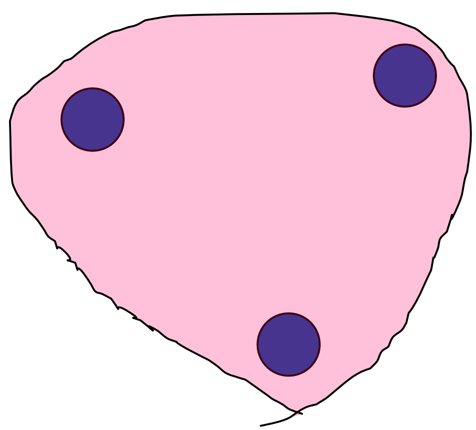
Fractional conflict graphs and conflict hypergraph



Input links



Fractional conflict graph
& independent sets



Conflict hypergraph

Approximation Results on MIS in SINR model

MIS has constant-factor approximations for:

- Uniform power in $\mathbb{R}^2$.
  [Goussevskaia,H,Wattenhofer,Welzl'09]
- Other fixed power in general metrics [H, Mitra, SODA'11]
- Arbitrary power control [Kesselheim, SODA'11]
  – Also, with power limitations [Wan'12, Kesselheim'12]
- Different fixed bit-rates [Kesselheim'12]
- Uniform power with spectrum sharing [H,Mitra'12]
  - with distributed learning [Asgeirsson, Mitra ,'11]
  - under jamming [Dams et al...]
- Holds also for an extension to Rayleigh fading
  [Dams, Hoefer, Kesselheim '13], [H, Tonoyan '17]

Intuition: Can ignore links with too much interference.

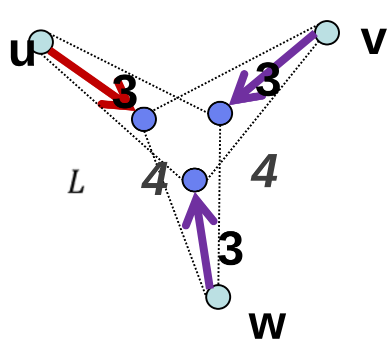
Scheduling in SINR model [in 2015]

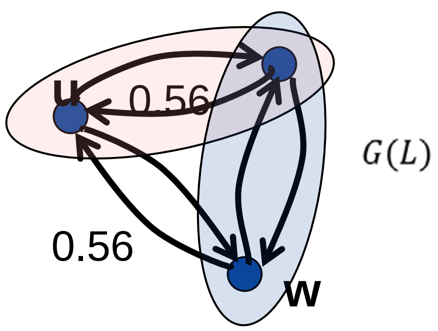
Scheduling (Coloring the conflict hypergraph):

- $O(\log n)$-approximation [Direct from MIS results]
- $O(\log \Delta)$-approximation (Δ = link length diversity)

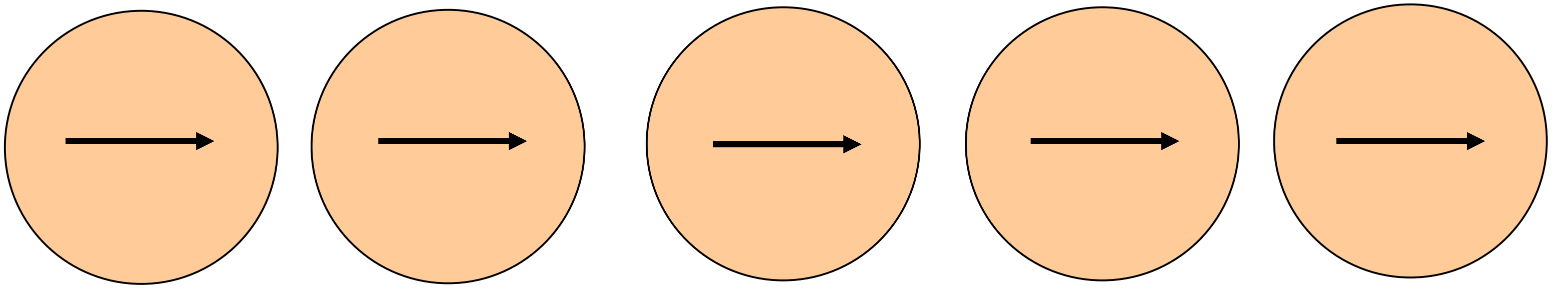
- Known algorithms give $\Omega(\log n)$-approximation [HKT '15]
  - None of the previously known techniques suffice to improve the performance guarantees

- $\Rightarrow$ Fractional graphs are harder than graphs
- $\Rightarrow$ Hypergraphs are harder than graphs





Equal-sized links

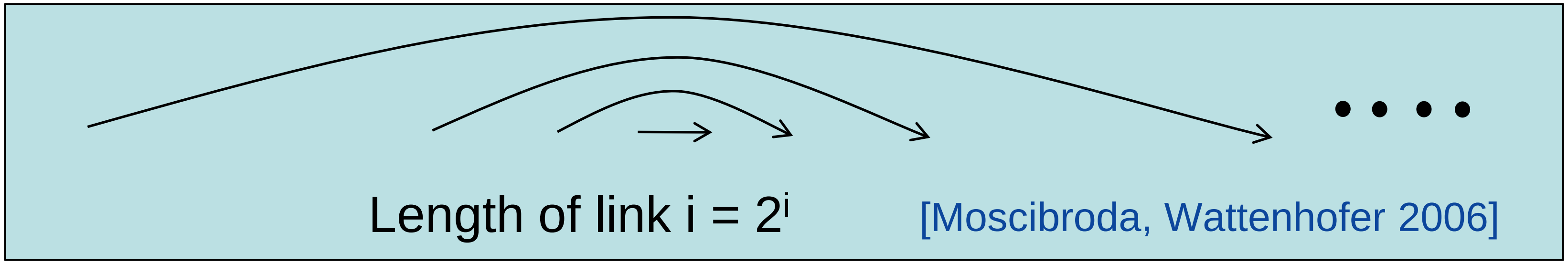


- If the links are evenly spaced, then the (relative) interference is a *converging geometric sum*
  [Goussevskaia, Oswald, Wattenhofer '06]

- Can model the links with *unit disks* (in an approximate sense) → *Unit disk graphs!*
  [H '09]

Disc Graphs natural for different-sized links

But they fail miserably!



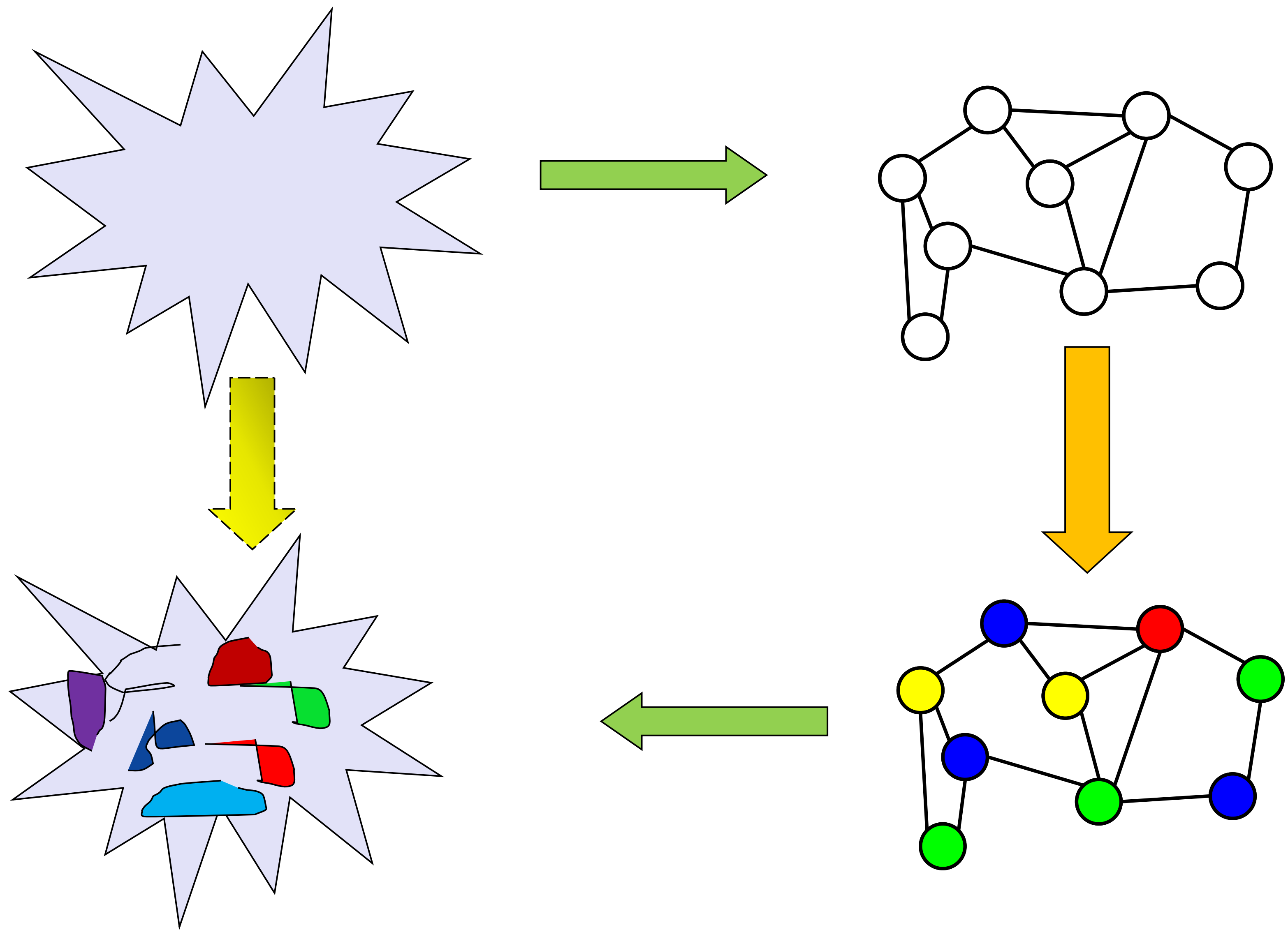
Feasible set but forms a clique in any disc graph

Core concept of this talk: Hypergraph sketches

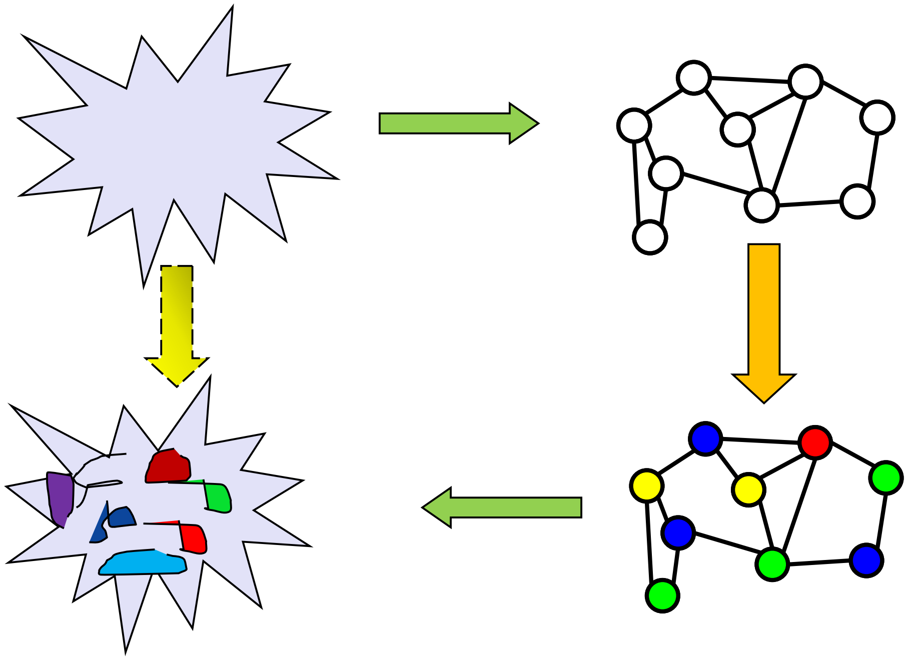
- Key question: If hypergraphs are so messy and hard to deal with, can we replace them with something easier?

- Can we replace them with ordinary graphs?

Approach: Abstract, solve, map back



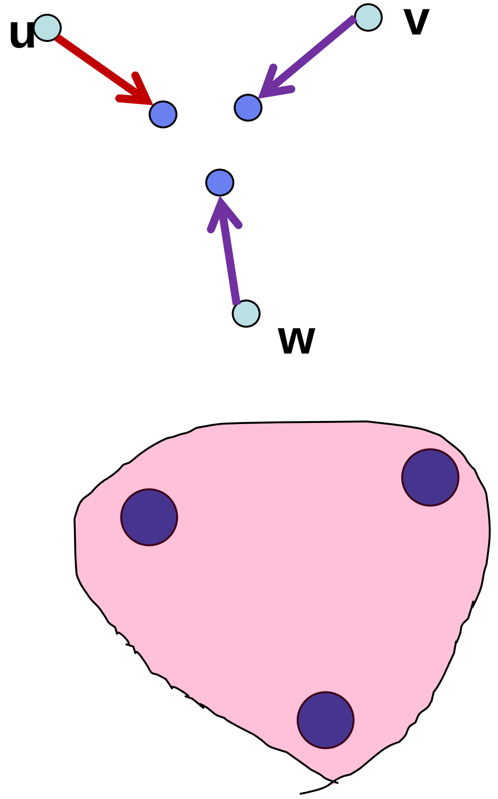
Price of abstraction

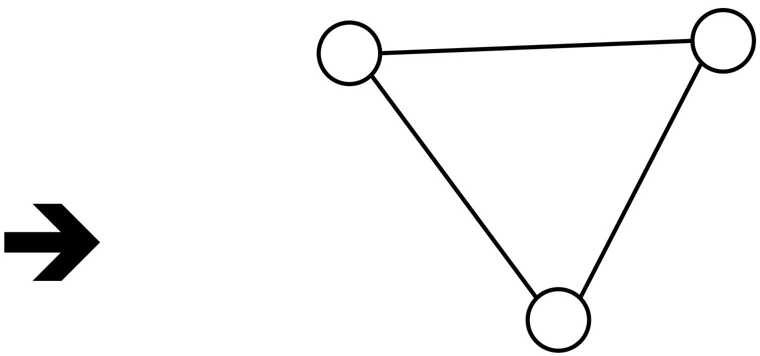


- Price of abstraction :

How much you lose by solving the abstracted problem (rather than solving directly)

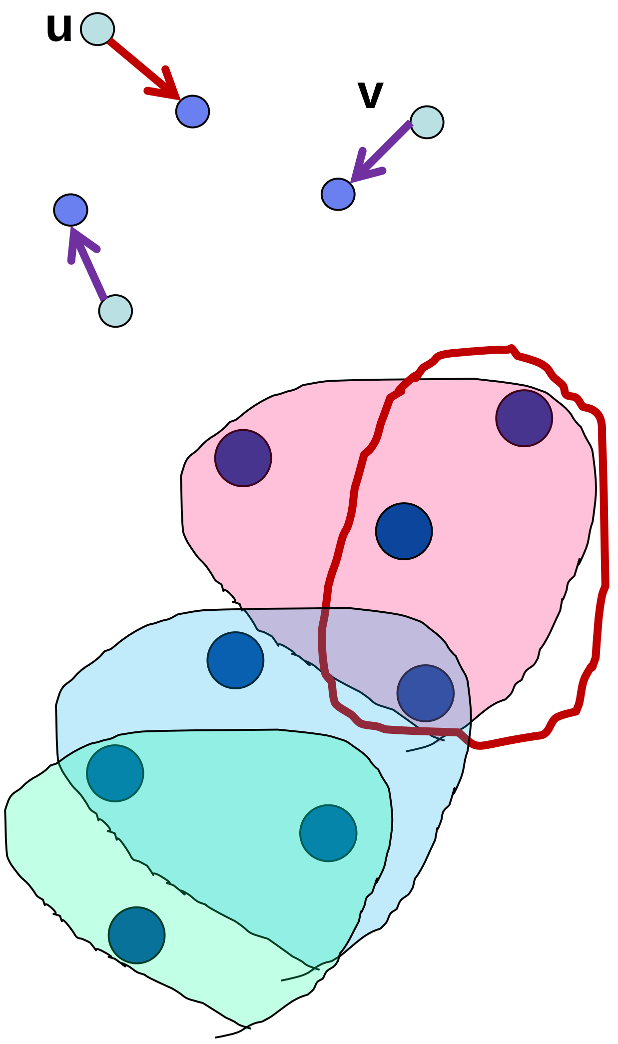
Representing link scheduling with a graph

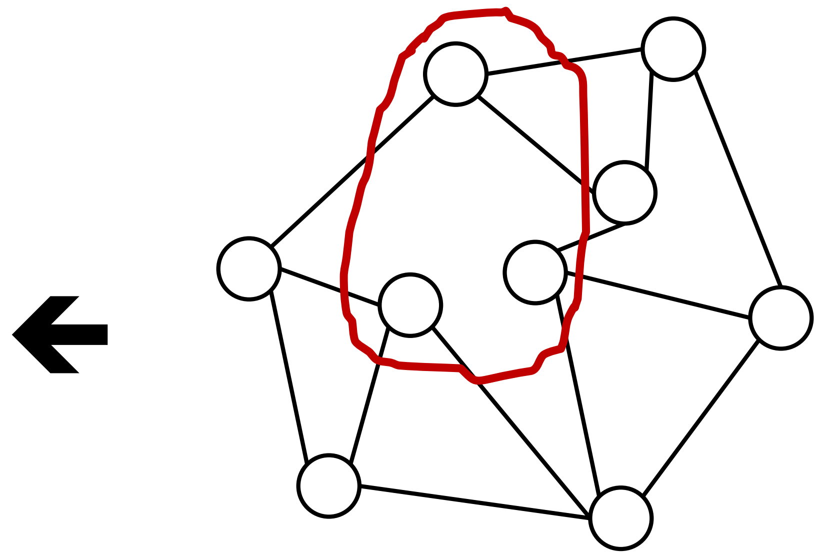




When should there be an edge?

Requirement I: Feasibility

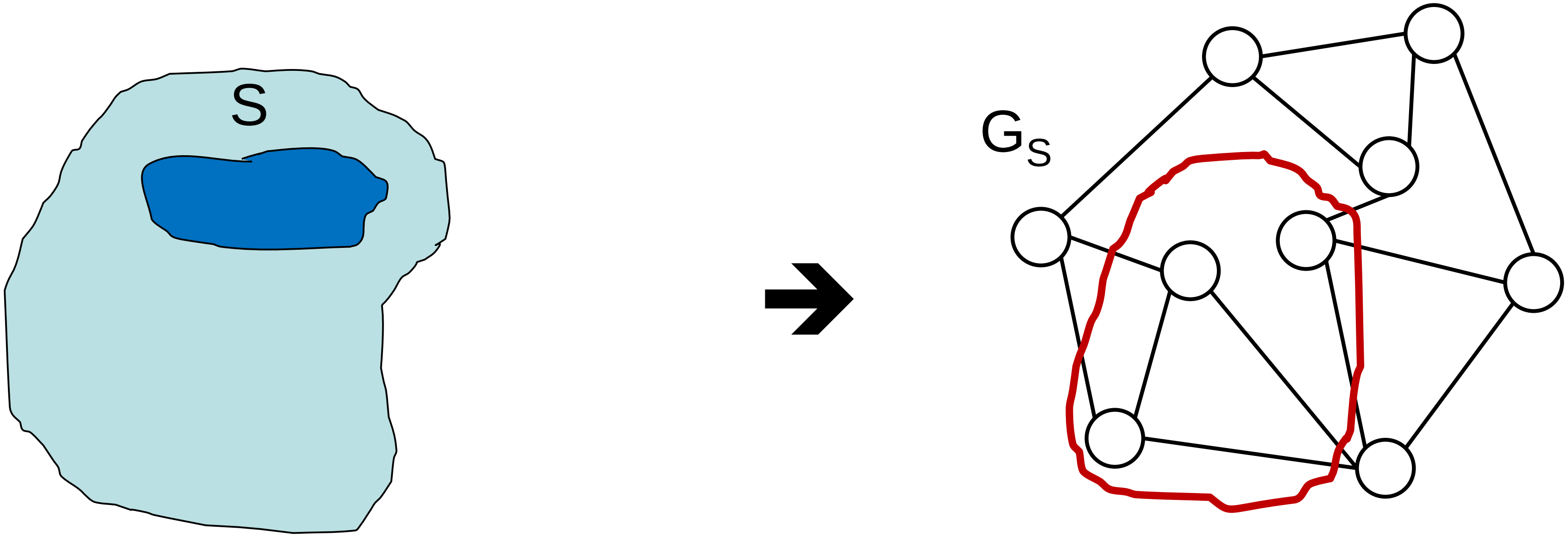




Independent sets should be <u>feasible</u>

valid coloring of G $\Rightarrow$ valid scheduling = valid coloring of H

Requirement II: Effectiveness



Feasible linksets should be „nearly independent“ in G

Small cost of abstraction!

S feasible $\Rightarrow \chi(G_S)$ small

Graphs sketching hypergraphs

- Given: Conflict hypergraph H
- Find: Schema to form a graph G s.t.

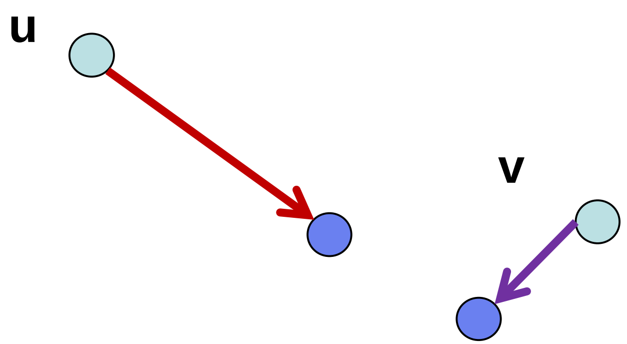
1. S is an independent set in G
   → S is independent in H
2. S is independent in H
   → $G(S)$ has low chromatic number, **k**

Cost of schema = *largest* value ***k***

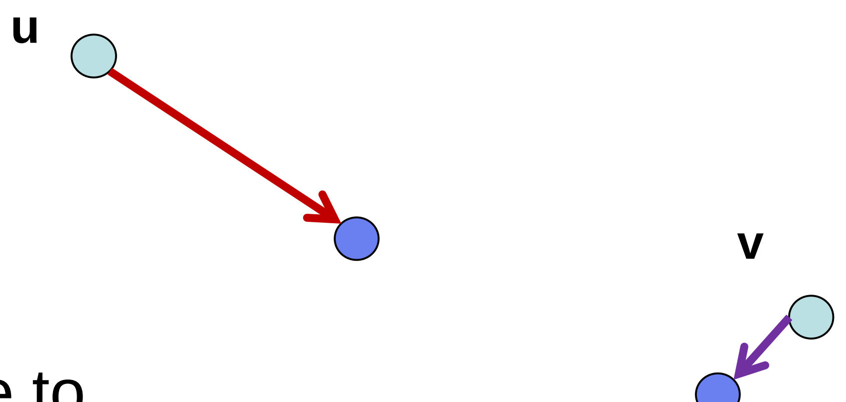
Price of graph abstraction = Minimum cost of a schema

Possible graphs schemas (that fail)

- Pairwise conflicts (the 2-edges of H)
  - The two links cannot coexist: One will always be infeasible
  - $d(u,v) \leq c \cdot \min(|u|,|v|)$
  - Too relaxed : fail feasibility



- Disc graphs
  - $d(u,v) \leq c \cdot \max(|u|,|v|)$
  - Too conservative
  - Only linear approximation



- Solution: Interpolate?
- - How strict do we have to be to maintain feasibility?

Conflict graph representations [H,Tonoyan, STOC'15]

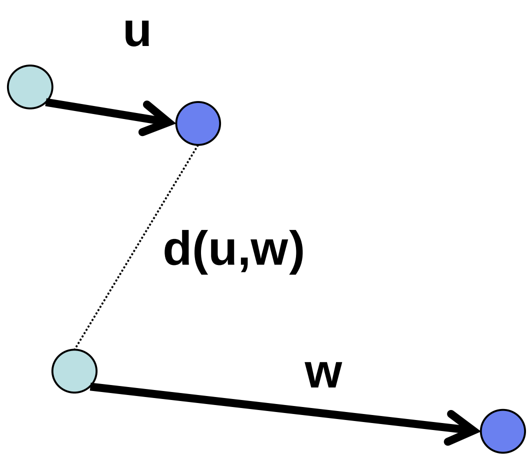
Adjacency predicate:

$$d(u,w) \leq f\left(\frac{|w|}{|u|}\right)|u|,$$

(f monotone)

f linear : disc graphs
f const : pairwise SINR

(w is longer than u)



All such graphs are O(1)-simplicial, which allows for constant-factor approximation of our problems

Conflict graph representations [H,Tonoyan, STOC'15]

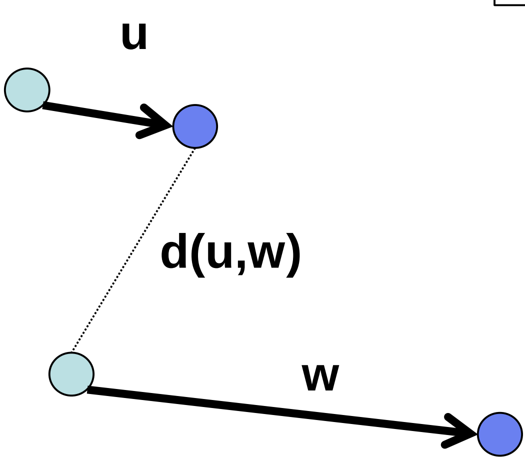
Adjacency predicate:

$$d(u,w) \le f\left(\frac{|w|}{|u|}\right)|u|,$$

(f monotone)

f linear : disc graphs
f const : pairwise SINR

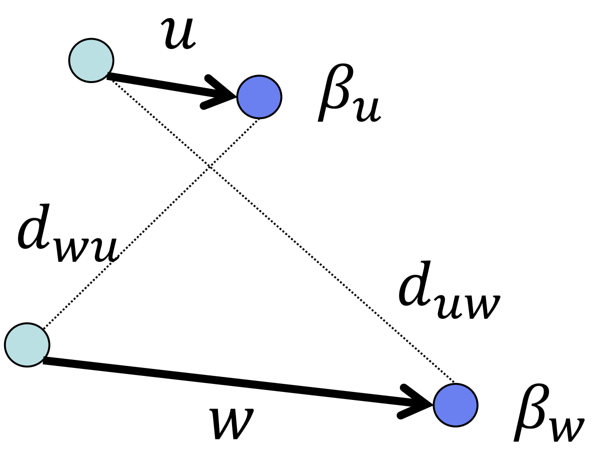
Feasibility holds for $f(x) = \Omega(\log x)$



Cost of abstraction is $f^*(x)$, the iterated application of f

For $f = \log$, the cost is $O(\log^* \Delta)$
Δ = Diversity in link lengths

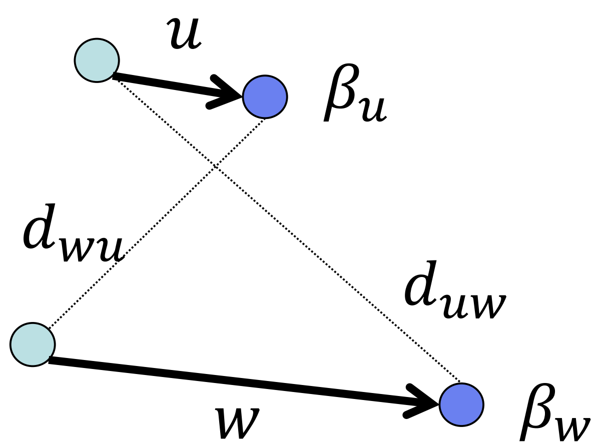
Conflict graph w/ data rates [H,Tonoyan ICALP'17]



High bit-rate means less error tolerance.
Requires smaller (relative) interference

Previous formulation failed to capture this

Conflict graph w/ data rates [H,Tonoyan ICALP'17]



Effective length of link u : $\ell_u = |u| \cdot \beta_u^{1/\alpha}$

Adjacency predicate:

$$d_{uw} d_{wu} \leq f\left(\frac{\ell_w}{\ell_u}\right) \ell_u \ell_w,$$

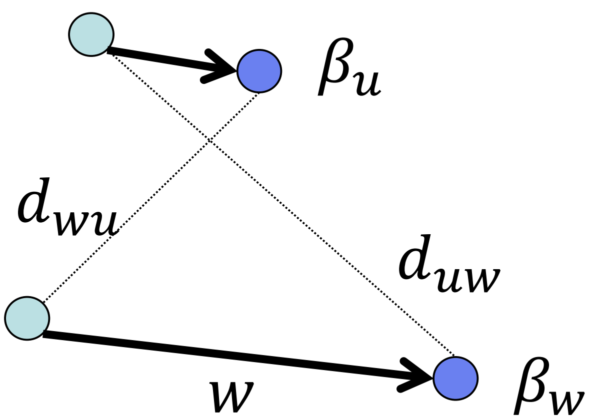
(for $\ell_u \leq \ell_w$)

Conflict graph w/ data rates [H,Tonoyan ICALP'17]

Adjacency predicate:

$$d_{uw} d_{wu} \leq f\left(\frac{|w|}{|u|}\right) |u||w|,$$

(f monotone)



Feasibility achieved with $f(x) = \sqrt[c]{x}$,
Price of abstraction $O(\log\log\Delta)$

Achievable with *oblivious* power: depends only on link length.

Stronger sandwiching property

Sandwiching property:

Given hypergraph H, we form *two* graphs G_1 and G_2 s.t.

For all subsets $S \subseteq V$,

$$\chi\big(G_1(S)\big) \leq \chi(H(S)) \leq \chi\big(G_2(S)\big)$$

and

$$\frac{\chi\big(G_2(S)\big)}{\chi\big(G_1(S)\big)} = O(\log\log\Delta)$$

Implications

- All major scheduling problems solved with $O(\log \log \Delta)$ factor, involving all available diversity
- Including:
  - Weighted MIS, and capacity region approximation
  - Different fixed rates, and rate control, with arbitrary utilities
  - Multi-channel multi-antennas variants
  - Multi-hop scheduling with fixed paths, or routing
  - Maximum (concurrent) multiflow,
  - Fractional scheduling
- Other applications:
  - Online algorithms (admission control, scheduling)
  - Spectrum auctions (additive, submodular, general valuations)
- Followup work: Connectivity & aggregation [MoWa06]

Limitation results for Conflict graph representations

- A. Necessarily of the form we consider $d(u,w) \leq f\left(\frac{|w|}{|u|}\right)|u|$
- B. Incur a $\Omega(\log\log\Delta)$ factor → Price of abstraction
- B‘. Incur a $\Omega(\log^*(\Delta))$ factor for uniform bit rates
- C. Cannot capture uniform power (= no power control)
- D. Requires Euclidean or doubling metrics
- E. No approximation in terms of *n* is possible.
- F. Requires interference-limited networks

Open questions :

- What other classes of hypergraphs have effective sketches?

- What can we say about this new class of graphs

Open questions

- Handling dynamic situations

- Uniform power
  - Only one power level
  - Should be „easier“, but we understand it less analytically

- Understanding SINR

- New modes of communication (*interference alignment*)
  - Beamforming, MIMO, *cooperative, cancellation*,...

Collaborators

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