

Algorithmic Glass Ceiling in Social Networks

The effects of social recommendations on network diversity

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Introduction

In this work, we study the effect of **social recommendations** on networks that exhibit community structure and homophily. We build models of network growth modeling natural and algorithmic growth under recommendations based on common neighbors. We find both empirically and theoretically that prominent social recommendation algorithms can **amplify** a pre-existing glass ceiling effect.

Glass Ceiling Effect

Definition. A graph sequence $G(n)$ for red (R) and blue (B) nodes exhibits a glass ceiling effect for the red nodes if

$$\lim_{n \rightarrow \infty} \frac{\text{top}_k(n)(R)}{\text{top}_k(n)(B)} = \infty \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{\text{top}_k(n)(R)}{\text{top}_k(n)(B)} = 0$$

where $\text{top}_k(R)$, $\text{top}_k(B)$ denotes the # nodes with degree $\geq k$.

Informally, this means that the minority of a network is under-represented at the top of the social hierarchy.

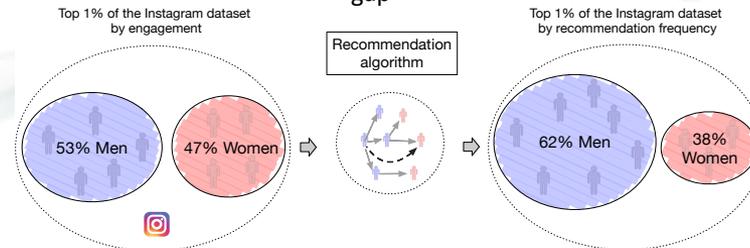
Results

Minority populations get further disadvantaged after adding recommendations

Glass ceiling effect is artificially amplified

Theorem: For a network with two populations, red (R) and blue (B) in which red nodes are the minority, for the graph sequences $G(n)$ for the organic growth model and $G'(n)$ for the recommendation model, the red and blue populations exhibit a power law degree distribution with coefficients:

$$\beta_{\text{rec}}(R) > \beta(R) > 3 > \beta(B) > \beta_{\text{rec}}(B)$$



Theory

Models of network evolution based on a bi-populated network (red and blue nodes):

- Fraction of red nodes = $r < 1/2$

At timestep t , a new edge is formed:

Organic growth model:

A new node connects naturally:

- η : randomly
- $1 - \eta$: preferential attachment

Recommendation model:

- ζ : organic growth (new node)
- $1 - \zeta$: existing node connects through a random walk of length 2

Homophily: if \neq labels, edge is accepted w.p. ρ

Competition

Key idea: Recommendations created through a random walk of length 2 reinforce **homophily**

Data

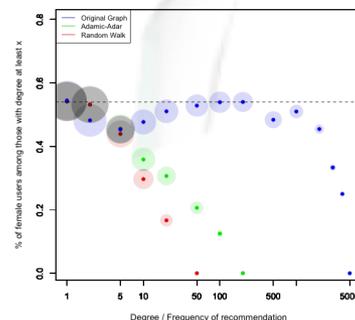
Data crawled from Instagram

- 54% women, 46% men
- Homophily: men are more inter-connected than women

Recommendation algorithms

simulated on Instagram data:

- Adamic-Adar
- Random Walk of length 2



Proof sketch & Conclusion

Wealth of red nodes:

- Fraction of edges towards R

$$\alpha_t = \sum_{v \in R} \text{indeg}(v) / t$$

Define a function F as the rate of growth of α_t

- F has a fixed point $\alpha \Rightarrow \alpha_t \rightarrow \alpha < r$

Organic growth

$$\alpha$$

Recommendation model

$$\alpha'$$

Key idea: Evolution equation \Rightarrow at equilibrium, the rate at which red edges appear = current fraction of red edges, as it does not evolve anymore.

Model dynamics of network growth:

- Preferential attachment \Rightarrow power law dynamics
- Homophily leads to the difference in the power law coefficients

A more inter-connected minority can reverse the glass ceiling effect!