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New Lower Bounds for the CONGEST Model

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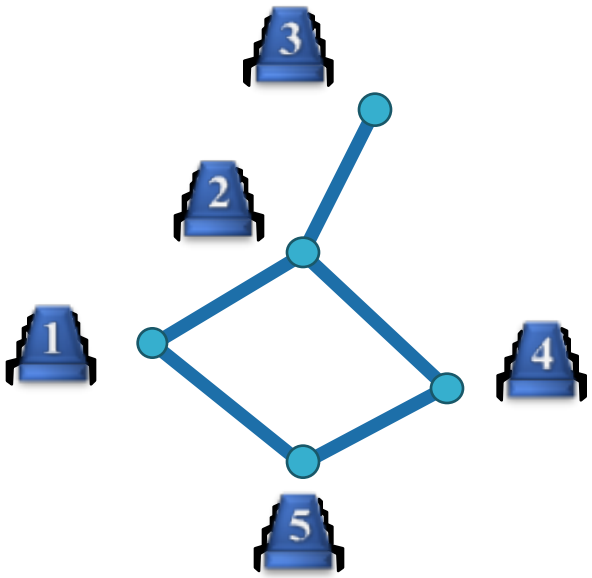
Joint work with Keren Censor-Hillel and Seri Khoury

The CONGEST Model

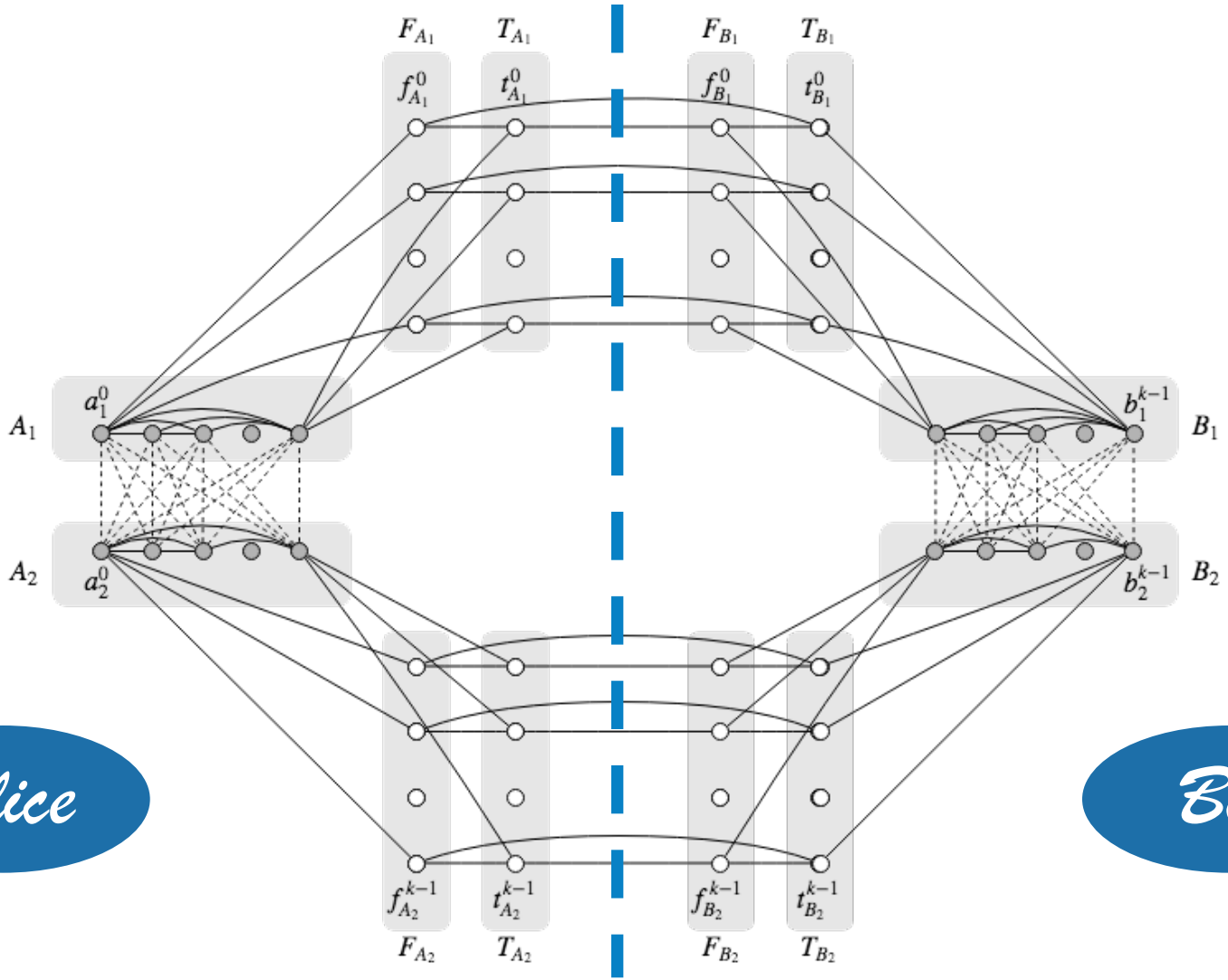
- $G = (V, E)$: network's topology
- n nodes = processors, unique ids
- Edges = communication links
  - $O(\log n)$-bit messages
- Synchronous communication
- Goal: minimum number of rounds



Problems & Complexities

$\Theta(n^2)$	• ?
$\Theta(n^\alpha)$	• ?
$\Theta(n)$	• ?
$\Theta(n/\log n)$	• Compute unweighted diameter
$\widetilde{\Theta}(\sqrt{n})$	• Build a MST
$\Theta(\log^* n)$	• 3-color a ring
$\Theta(1)$	• Verify coloring

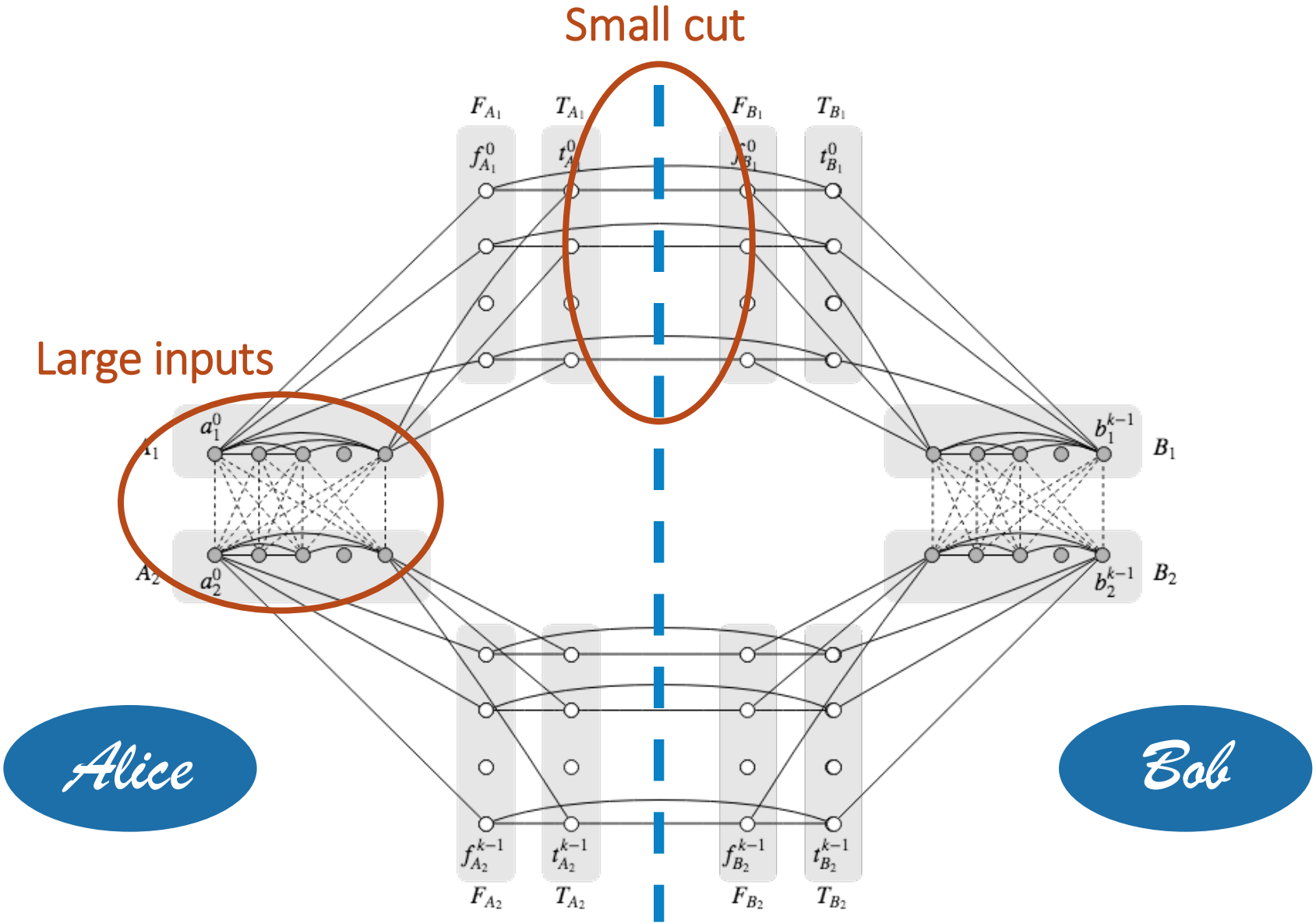
MVC Lower Bound



Alice

Bob

MVC Lower Bound

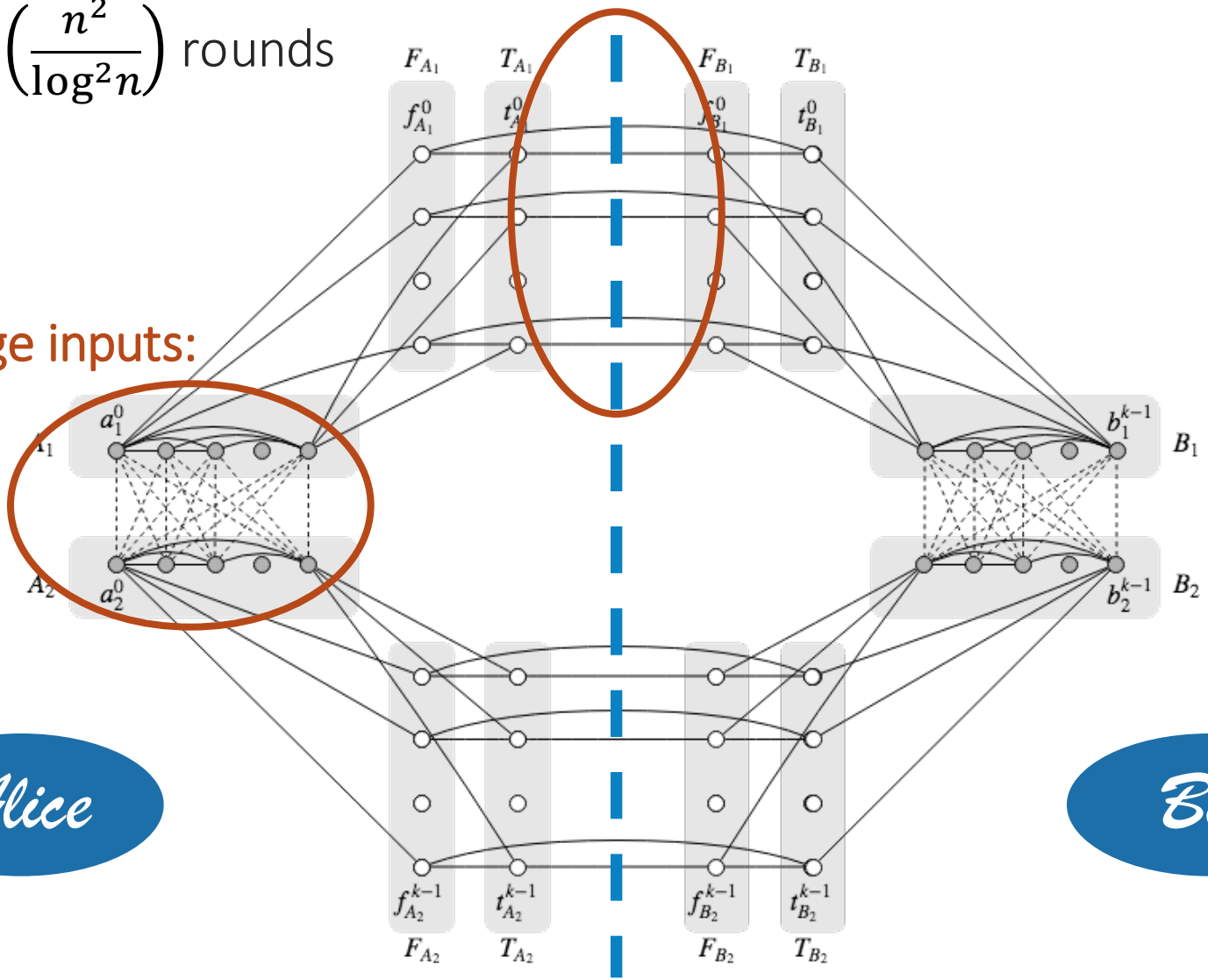


MVC Lower Bound

- $\Omega\left(\frac{n^2}{\log^2 n}\right)$ rounds

Small cut: $\log n$

Large inputs: n^2



Alice

Bob

Results I

$\Omega\left(\frac{n^2}{\log^2 n}\right)$ lower bound for:

- Minimum vertex cover
- Maximum independent set
- Computing the chromatic number χ
- Weighted cycle detection

Results II

- $\Omega(n)$ lower bound for weighted APSP
  - Unweighted APSP solvable in $O(n/\log n)$ [HFQ+16]
- Alice and Bob can solve weighted APSP in linear time
- $\Omega(n^2)$ lower bound for identical subgraphs – deterministic
- $O(1)$ algorithm for identical subgraphs – randomized

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Problems & Complexities

$\widetilde{\Theta}(n^2)$	• MaxIS, MVC, χ-coloring, cycle detection
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Weighted APSP?

Problems & Complexities

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Weighted APSP?

Waiting for you at the poster session!