

On the Complexity of Closest Pair via Polar-Pair of Point-Sets

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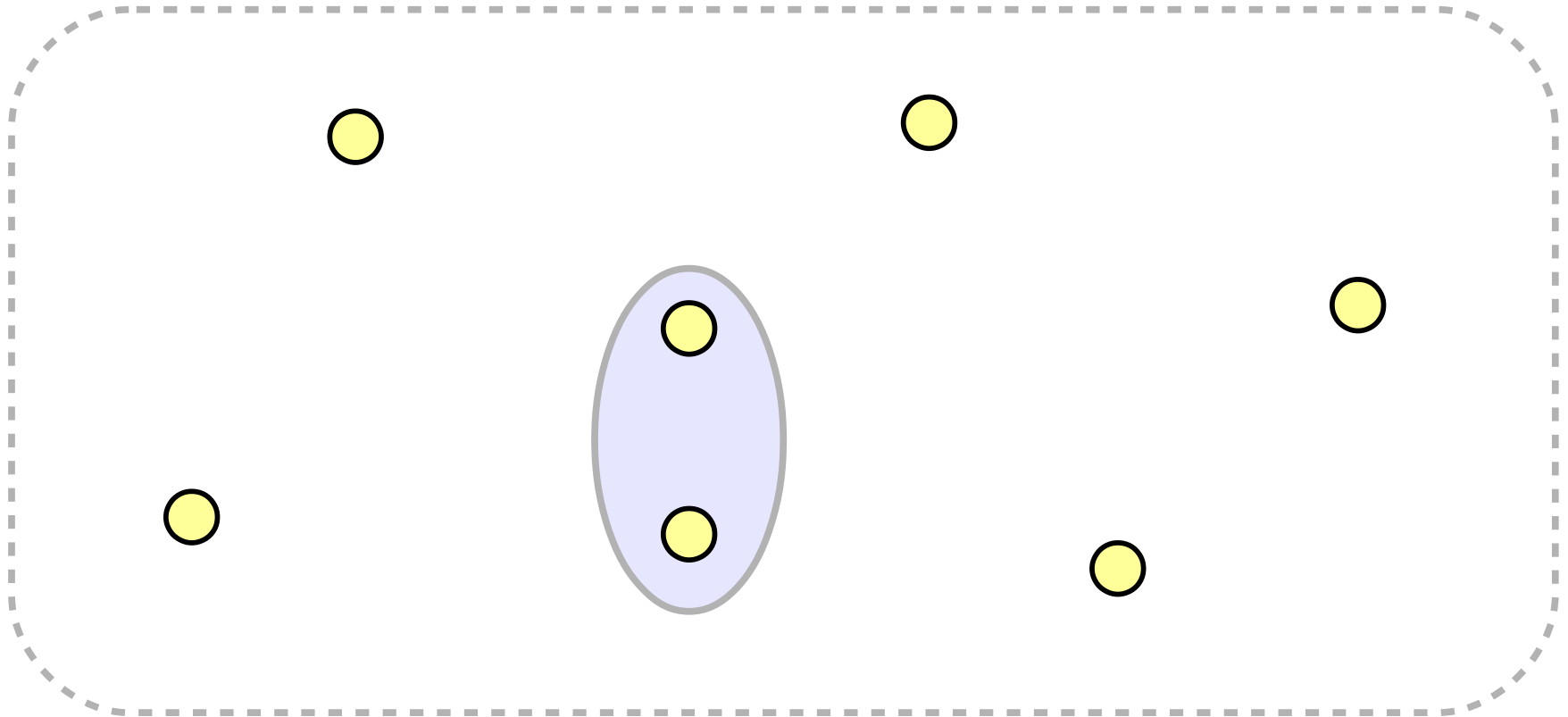
Joint work with Roei David and Karthik CS

This Talk

- Complexity of Closest Pair
- Geometric Representation of Graphs

Closest Pair (CP)

Given a collection of n points in a d -dimensional metric, find a pair of points with minimum-distance.



Known Algorithms

- Euclidean Closest Pair
 - Dimension $d=O(1)$:
 - $O(2^D n \log n)$ (deterministic) [Bentley-Shamos'76]
 - $O(2^D n)$ (randomized) [Rabin'76, Khuller-Mattias'95]
 - Dimension $d=\Theta(\log n)$
 - $O(d n^2)$ (trivial algorithm)
 - Dimension $d=n$: $O(n^{3-\epsilon})$, for some $\epsilon > 0$

Is there an $O(n^{1.9})$ -time algorithm
when dimension $d = \Omega(\log n)$?

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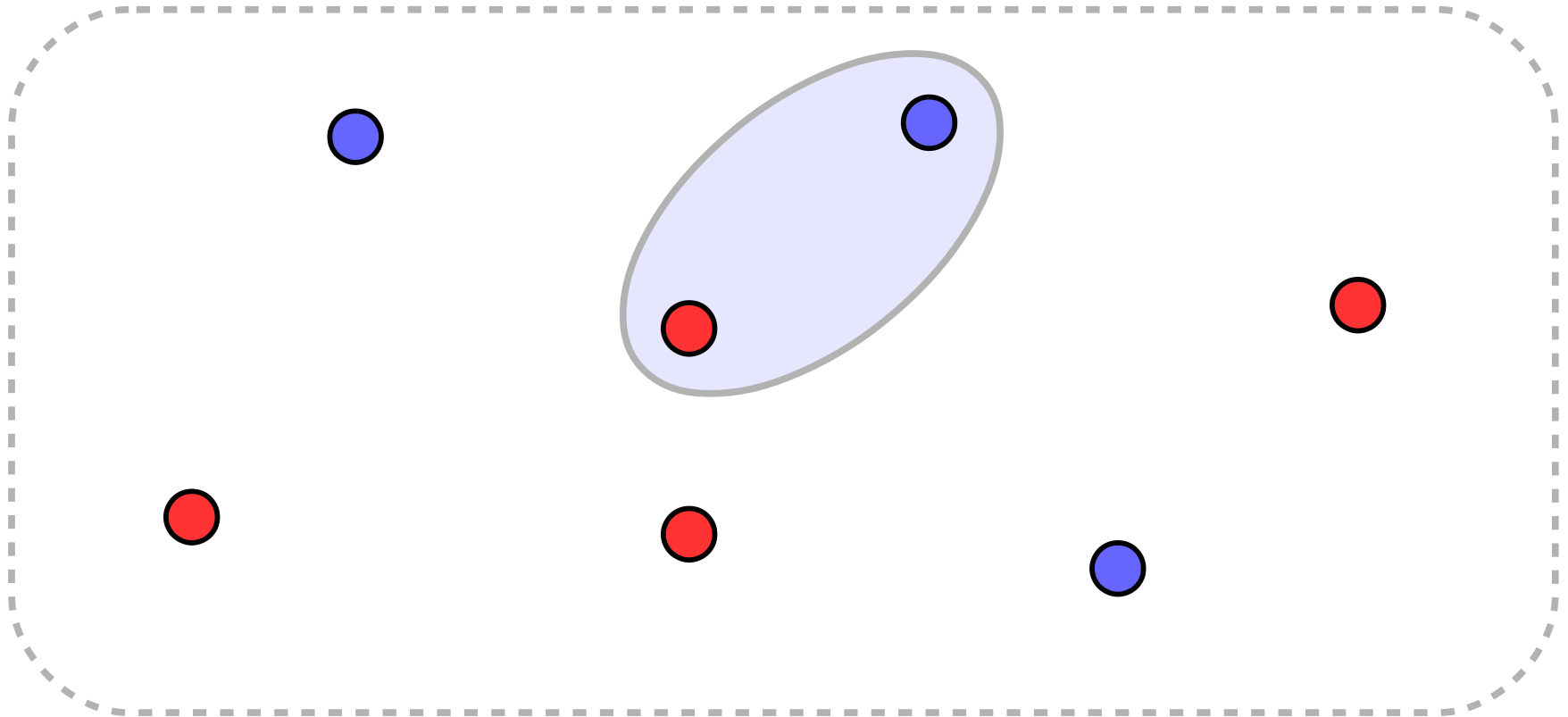
Don't know for Euclidean Closest Pair.

No for the **bichromatic** variant.

[Alman-Williams 2015]

Bi-Chromatic Closest Pair (BCP)

Given a collection of n **red** and n **blue** points in a d -dimensional metric, find a pair of **red-blue** points with minimum-distance.

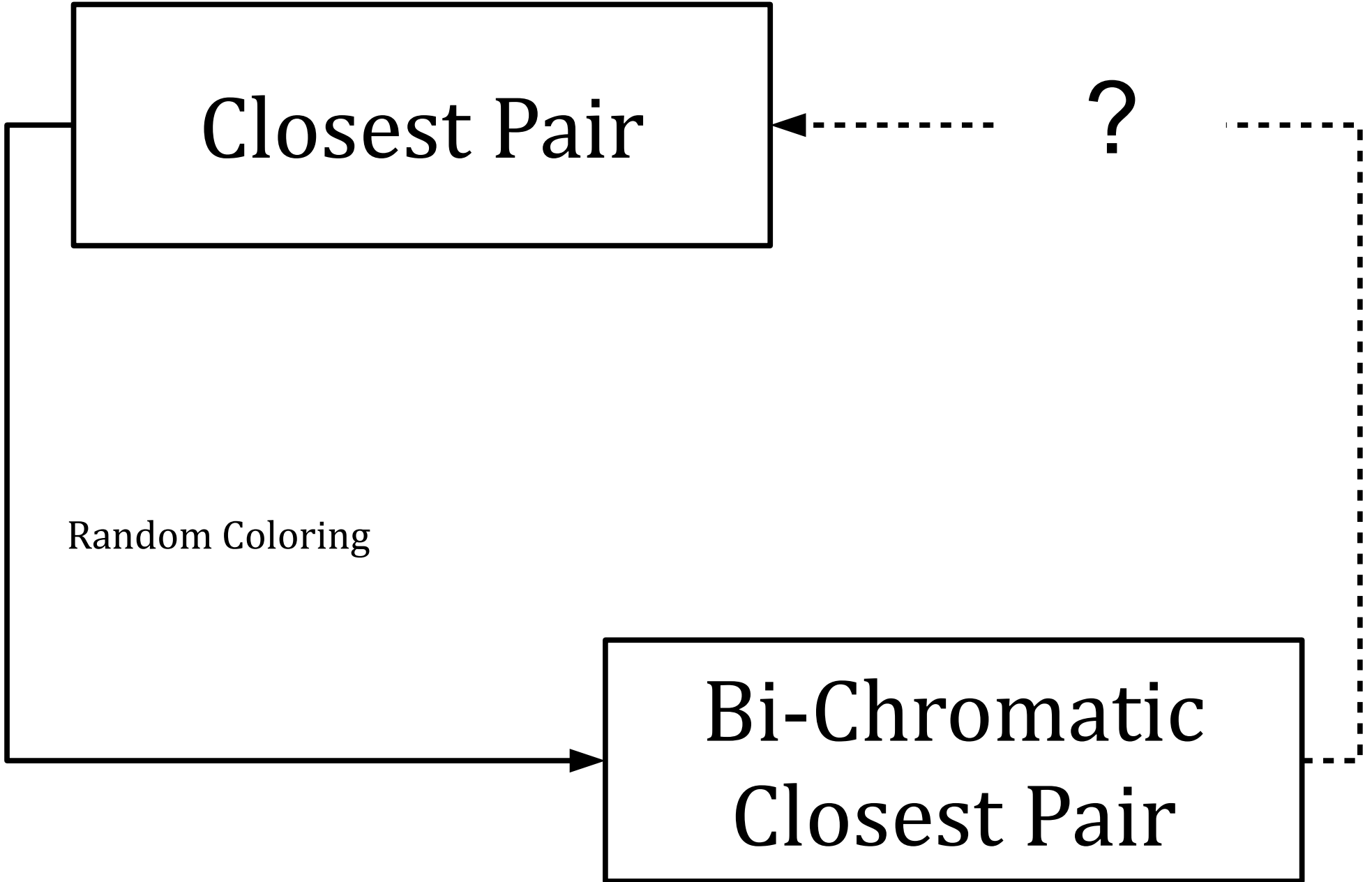


Closest Pair

?

Random Coloring

Bi-Chromatic
Closest Pair



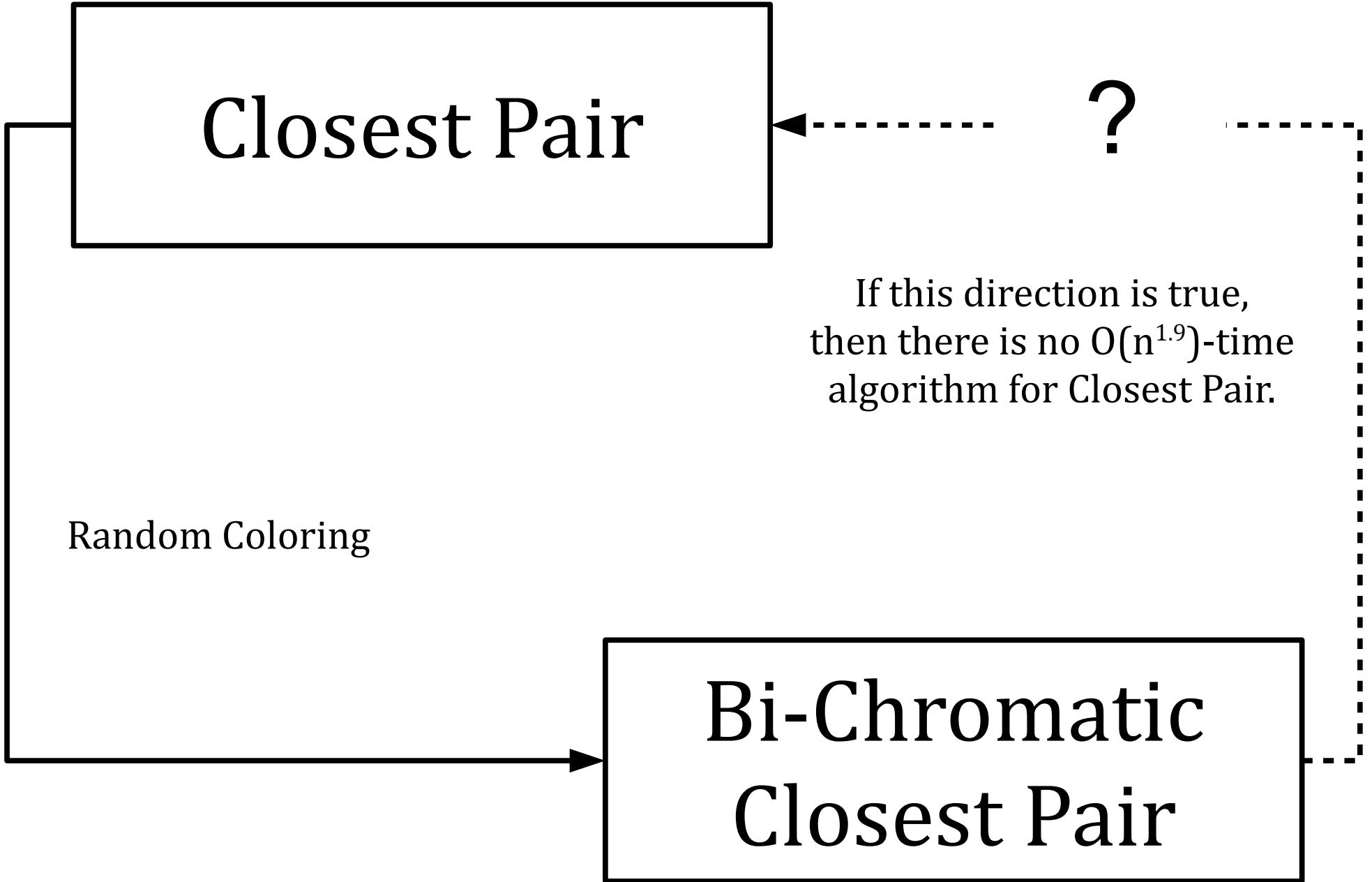
Closest Pair

?

If this direction is true,
then there is no $O(n^{1.9})$ -time
algorithm for Closest Pair.

Random Coloring

Bi-Chromatic
Closest Pair



Closest Pair

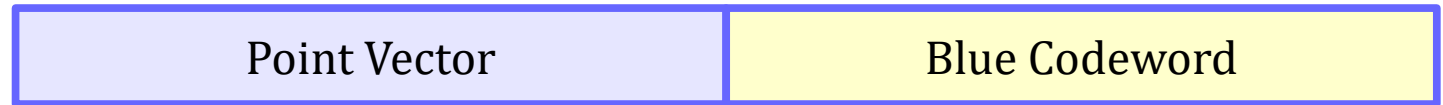
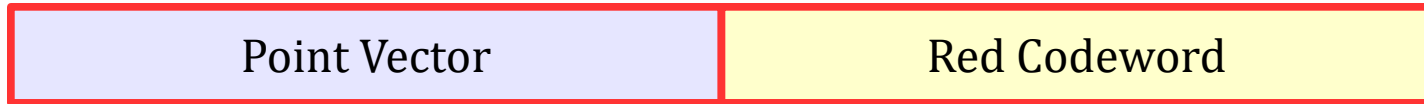
Exists for L^p -metrics for $p > 2$
via random codes.

Random Coloring

Bi-Chromatic
Closest Pair

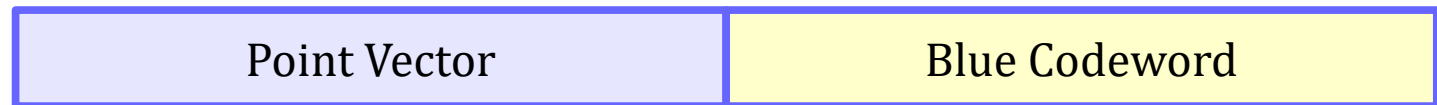
Reduction BCP \rightarrow CP

Concatenate point-vectors with codewords



Reduction BCP \rightarrow CP

Concatenate point-vectors with codewords



Needed Properties of The Codewords

(Bi-Clique Property)

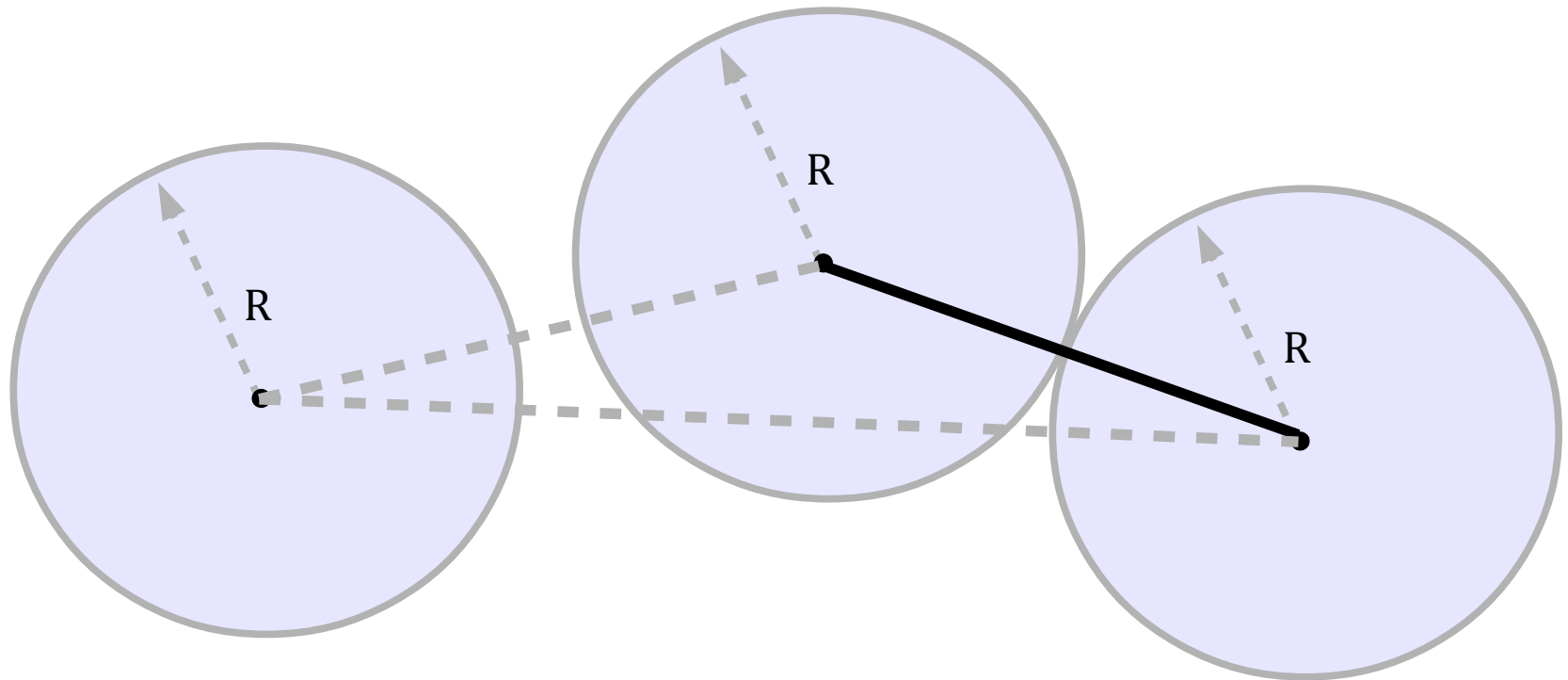
$$\text{Distance}(\text{Red-Code}, \text{Red-Code}') \geq R + 1/n$$

$$\text{Distance}(\text{Blue-Code}, \text{Blue-Code}') \geq R + 1/n$$

$$\text{Distance}(\text{Red-Code}, \text{Blue-Code}) = R$$

The existence of Codewords with
Bi-Clique Property implies $\text{BCP} \rightarrow \text{CP}$
(that runs in $O(n^{1.9})$ -time)

Complexity Question of CP reduces to Geometric Representation of Bi-Clique



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What is the smallest dimension to
represent a bi-clique in L^p -metric?

(contact-dimension of bi-clique (bicd))

CP & BCP are equivalent in
 $O(\log n)$ -dimension L^p -metrics (for $p > 2$)

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How about other L^p -metrics?

Known Bounds

for Bi-Clique Contact Dimension

	n	$\leq \text{bicd}(L^0)$	\leq	n	
	?	$\leq \text{bicd}(L^1)$	\leq	n^2	
	?	$\leq \text{bicd}(L^p)$	\leq	n	for $1 < p < 2$
Maehara 1985 Frankl-Maehara 1988	1.286 n	$\leq \text{bicd}(L^2)$	\leq	1.5 n	
	$\Omega(\log n)$	$\leq \text{bicd}(L^p)$	\leq	$O(\log n)$	for $p > 2$
	$\Omega(\log n)$	$\leq \text{bicd}(L^\infty)$	\leq	$2 \log_2 n$	

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$1.286 n$	$\leq \text{bicd}(L^2)$	$\leq 1.5 n$	
$\Omega(\log n)$	$\leq \text{bicd}(L^p)$	$\leq O(\log n)$	for $p > 2$
$\Omega(\log n)$	$\leq \text{bicd}(L^\infty)$	$\leq 2 \log_2 n$	

David, Karthik CS, L. 2018

Bi-Clique has no contact-graph
in $O(\log n)$ -dimension for most L^p -metrics.

Open Problems

- What is the lower bound for $\text{bicd}(L^1)$?
 - Related to **Kusner's conjecture** on equilateral dimension of L^1
 - For clique, Alon-Pavel shows $\text{contact-dim} \geq \Omega(n / \log n)$
- Better Lower / Upper Bounds for L^1 and L^2 ?
 - An alternative way to reduce $\text{BCP} \rightarrow \text{CP}$?
 - One will need a **white-box** reduction.

The End

Thank you for your attention.

Questions?

The End

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Karthik C.S. will give a 20 min talk in SoCG'18 next week.