Approximating Geometric Knapsack via L-Packings

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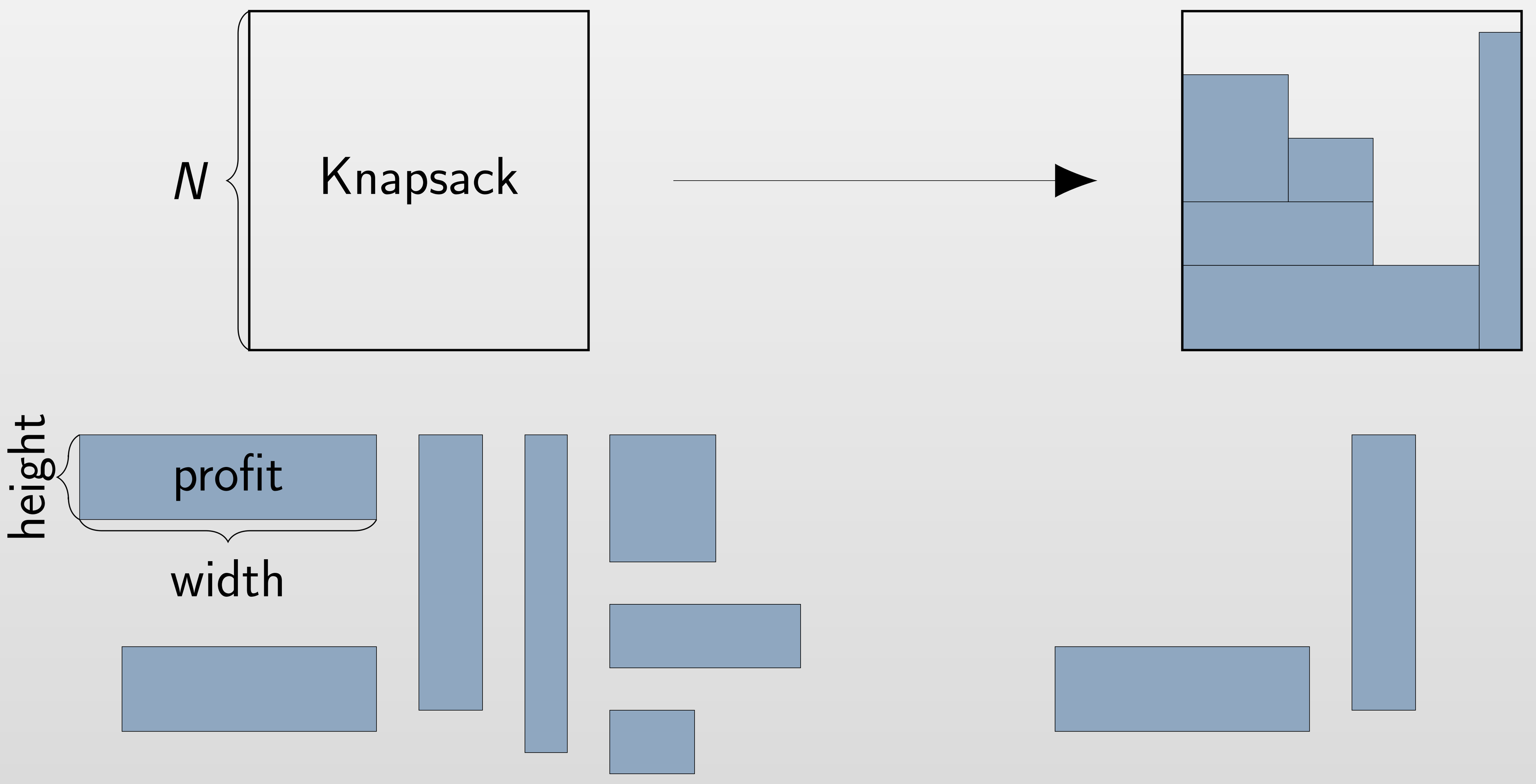
Definition of the problem

An instance of Two-Dimensional Geometric Knapsack consists of:

- A set of n items represented by axis-parallel rectangles, each one characterized by an integral height, width and profit;
- A knapsack of dimension $N \times N$, with $N \in \mathbb{N}$.

The goal is to find a non-overlapping packing of a subset of items that maximizes the total profit.

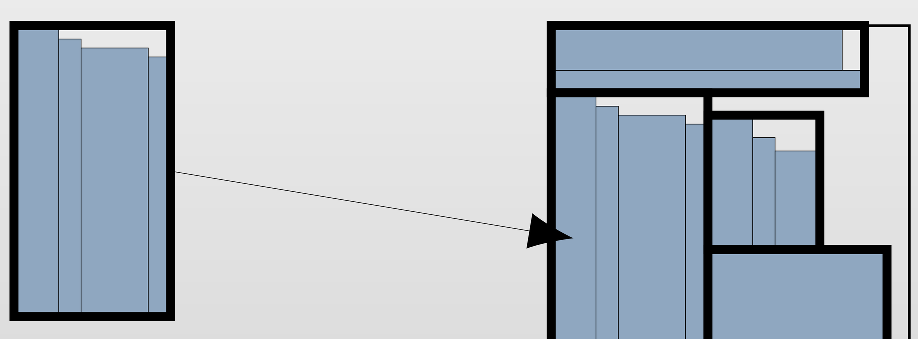


Variants and Known Results

The following variants of the problem have been studied:

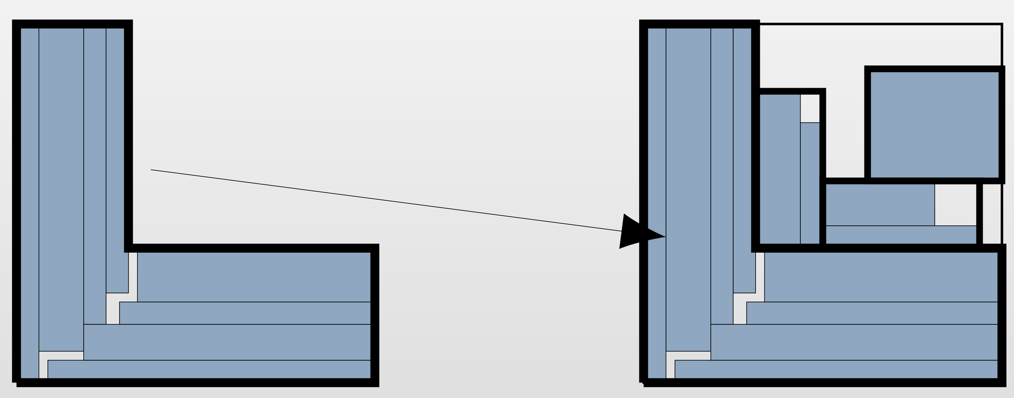
- When all the profits are equal to 1 (Cardinality case);
- When items can be rotated by 90 degrees.

For each possible variant the best known result was a $(2+\varepsilon)$-approximation, based on **Container Packings**, which can be computed almost optimally in polynomial time via Dynamic Programming.



A Different Approach: L-packing

We deviate from the previous approach and now decompose the solution into containers **plus an L-packing**. One of the main contributions of our work is a $(1+\varepsilon)$-**approximation for L-packing instances**.

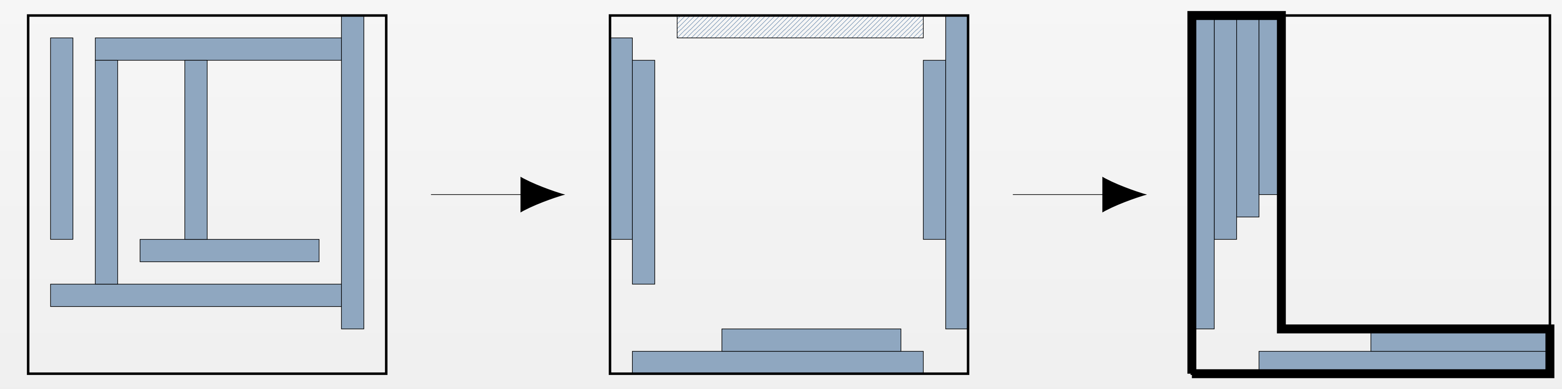


A simple $\left(\frac{16}{9}+\varepsilon\right)$-approximation for the Cardinality case without Rotations.

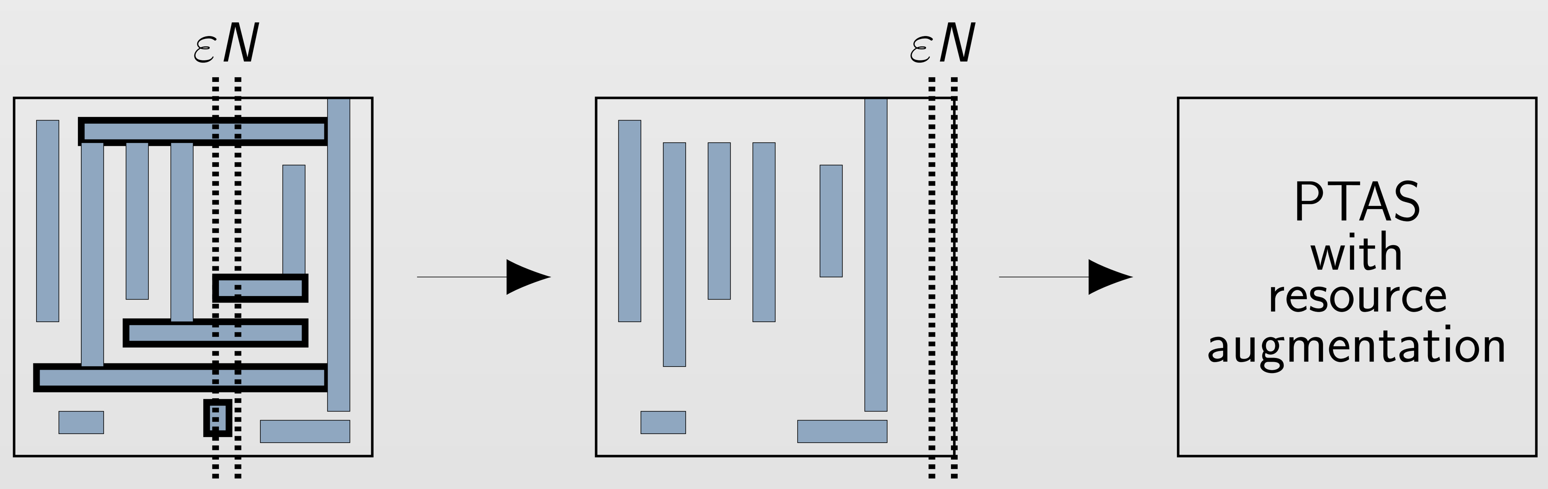
Let $\varepsilon > 0$ and let OPT be the optimal solution. Since the number of items with both dimensions larger that εN in OPT is constant, we can safely discard them.

Let L denote the set of items with one dimension strictly larger than $\frac{1}{2}N$ and S the set of remaining items.

- First packing of total profit $\left(\frac{3}{4}-\varepsilon\right)|L \cap OPT|$.



- Second packing of total profit $\left(\frac{1}{2}-\varepsilon\right)|L \cap OPT| + \left(\frac{3}{4}-\varepsilon\right)|S \cap OPT|$.



The probability of intersecting a rectangle in L is at most $\frac{1}{2}+\varepsilon$ and of intersecting a rectangle in S is at most $\frac{1}{4}+\varepsilon$.

Taking the best out of the two solutions we get a $\left(\frac{16}{9}+\varepsilon\right)$-approximated solution.

Final Results.

The described approach can be refined and combined with additional techniques, obtaining the following results for the different cases:

- A $\left(\frac{17}{9}+\varepsilon\right)$-approximation for the weighted case without rotations;
- A $\left(\frac{558}{325}+\varepsilon\right)$-approximation for the cardinality case without rotations;
- A $\left(\frac{3}{2}+\varepsilon\right)$-approximation for the weighted case with rotations; and
- A $\left(\frac{4}{3}+\varepsilon\right)$-approximation for the cardinality case with rotations.

Open Questions.

- How to solve multiple L-packing instances simultaneously?
- How to solve a **Ring-packing** instance? Or a more general **Corridor-packing** instance?
- How to solve L-packing instances when rotations are allowed?