99% Revenue via Enhanced Competition

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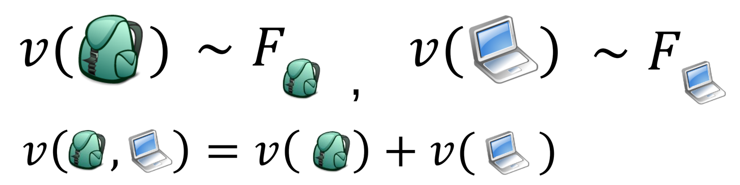
MOTIVATION

Single item, n buyers:

$v_i \sim F_i$ Myerson's mechanism (81) is optimal. For one buyer: price at $\max_p p \cdot \Pr[v \geq p]$

Many additive items: Hard even for one buyer.



Optimal Mechanisms: Require randomization, Revenue Non-monotonicity, Infinite menu size, Hard to find [HR 15, HN 13, DDT 14]

Research Agenda 1: Approximation

*How much revenue can simple mechanisms **guarantee**, compared to the (impractical, unachievable) optimal revenue?*

Simple Mechanisms:

1. *Rev* – Optimal (impractical, unachievable) revenue.
2. SRev – Optimal revenue from selling each item separately.
3. BRev – Optimal revenue from selling the grand bundle.
4. VCGE – VCG with entry fees.

Previous Results [HN12, CH13, LY13, BILW14, CDW16]:

One Buyer	Many Buyers
$Rev \leq O(\log \#items) \cdot SRev$	$Rev \leq O(1) \cdot \max\{BRev, VCGE\}$
$Rev \leq O(1) \cdot \max\{BRev, SRev\}$	$Rev \leq 0.99 \cdot SRev$ *IID, MHR

Research Agenda 2: Enhanced Competition

*How much more **competition** is required, for simple mechanisms to **exceed** the (impractical, unachievable) optimal revenue?*

Previous Results:

- n I.I.D. buyers, m items, regular distributions.
- VCG(x) – revenue from VCG with x buyers.

[BK96]: m=1:	*Rev*(n) ≤ VCG(n+1)
[RTY 12]: unit-demand buyers:	DET-*Rev* (n) ≤ VCG(n+m)
[EFFTW17]: additive buyers	*Rev*(n) ≤ VCG(O(n+m))

This work: Combine agendas

*How much more **competition** is required, for simple mechanisms to **almost exceed** the (impractical, unachievable) optimal revenue?*

MAIN MESSAGE

99% Rev requires much less competition!

MODEL

- m item distributions: $F_1, \ldots, F_m$
- n buyers at first, buyer j's value for item i: $v_i^j \sim F_i$
- Values drawn independently, and $v^j(S) = \sum_{i \in S} v_i^j$
- Truthfulness: Seller selects a Bayesian Individually Rational and Incentive Compatible mechanism M.
- M = (allocation function, payment function)
- Objective: Maximize expected total payment
- $Rev(n)$ = Optimal revenue from n i.i.d. buyers.

UPPER BOUNDS

$$99\% \; Rev(n) \leq SRev\left(O\left(n \cdot \log \frac{m}{n}\right)\right)$$

If $n \gg m$:

$$99\% \; Rev(n) \leq SRev(n)$$

If $n = 1$:

$$99\% \; Rev(1) \leq \max\{BRev, SRev\}\,(O(1))$$

If distributions are **regular**:

$$99\% \; Rev(1) \leq BVCG(O(1))$$

BVCG = second price for the grand bundle.

LOWER BOUNDS

If $m \gg 1$ and $\frac{m}{n} > CONST$, there are instances where

$$SRev\left(O\left(n \cdot \log \frac{m}{n}\right)\right) \ll Rev(n)$$

COMPARISON TO PREVIOUS WORK

Upper bounds:

When $m \gg n$:

- [EF**F**TW17] showed for **regular distributions**: $Rev(n) \leq VCG(n+m)$, we improve $O(m)$ to $O\left(n \cdot log \frac{m}{n}\right)$, at a loss of 1% revenue.

When $n \gg m$:

- [CH13] showed 99% $Rev(n) \leq SRev(n)$ for n larger than some constant, for **MHR distributions**, our result is an analogue for **general distributions**.
- [EF**F**TW17] showed 50% $Rev(n) \leq VCG(n)$ for **regular distributions**, we improve to 99%.

When $n = 1$:

- [GK16] shows Random(sell separately, sell bundle) using a single sample guarantees a constant fraction of Rev. Our result implies: "sell bundle for the price of a single sample" suffices.

Lower bounds:

Generalizes a result for $n = 1$ ([EF**F**TW17]).

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