Lower Bounds for Symbolic Computation on Graphs

Highlights of Algorithms 2018

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Based on:
Krishnendu Chatterjee, W. Dvořák, Monika Henzinger, and Veronika Loitzenbauer.
Lower Bounds for Symbolic Computation on Graphs: Strongly Connected Components, Liveness, Safety, and Diameter. SODA 2018, pp. 2341-2356

Motivation

- Graph algorithms are central in the analysis of reactive systems:
  - States of the System → Vertices of the graph
  - State Transitions → Edges of the graph
- The **resulting graphs are huge**
  - The number of vertices is exponential in the number of variables
  - Explicit representation of graphs is infeasible
  - Graphs are implicitly represented using e.g. binary-decision diagrams (BDDs) = symbolic computation
- **Set-based symbolic model of computation**
  - Same operations as standard RAM algorithms, except
    - for access to the edges and nodes of the input graph
    - for manipulation of sets of vertices

Model: Set-based Symbolic Computation

- Access to edges: Only through **One-step operations Pre and Post:**
  - Predecessor Operation *Pre*(X):

$$\boldsymbol{Pre(X) = \{\, v \in V \mid \exists x \in X : (v, x) \in E \,\}}$$

  - Successor Operation *Post*(X):

$$\boldsymbol{Post(X) = \{\, v \in V \mid \exists x \in X : (x, v) \in E \,\}}$$

- Manipulation of sets of vertices: ***Basic set operations***
  - Given one or two sets of vertices, we can perform basic set operations like union, intersection or complement
- **Symbolic Space requirement** = number of sets simultaneously stored by an algorithm
  - We deal with compact representation of huge graphs
  - The number of stored sets should be small w.r.t. size of the graph, ideally constant.

Fundamental problems in graphs

- *Problems on graphs:* Starting from a vertex we have to decide whether there exists an infinite path satisfying a certain objective.
  - Objectives arising in the analysis of reactive systems:
    - Reachability
    - Safety
    - Liveness (Büchi)
    - co-liveness (coBüchi)
- Computing SCCs is at the heart of the fastest algorithms for the above problems (and thus of interest)
- Many graphs, e.g., in hardware verification, have small diameter D which (once detected) can be exploited for more efficient algorithms
  - We consider computing the (approximate) diameter of a graph

Results

- First lower bounds for the Set-based Symbolic Model of Computation
  - Demonstrate Communication Complexity to be an appropriate tool to show lower bounds for symbolic computation
- Matching upper and lower bounds for fundamental problems

Reach	SCC	Safety	Büchi	coBüchi
$\Theta(D)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$

- Interesting gap between Reach $\Theta(D)$ and the other problems $\Theta(n)$ even for constant diameter graphs

Results

- Refined Analysis of the SCC algorithm by Gentilini et al.
  - and matching lower bounds
    - $\Theta(\min(n, (D \cdot |SCCs(G)|), (\sum_{C \in SCCs(G)} (D_C + 1)))$
    - D ... Diameter of the Graph
    - D_C ... Diameter of the SCC C

- Upper and lower bounds for (approximate) diameter

	exact	$(1+\epsilon)$ approx	$(3/2-\epsilon)$ approx	2 approx
Upper bound	$O(n \cdot D)$	$\tilde{O}(n \cdot \sqrt{D})$	$\tilde{O}(n \cdot \sqrt{D})$	$O(D)$
Lower bound	$\Omega(n)$	$\Omega(n)$	$\Omega(n)$	

Summary and Conclusion

- Different model of computation: **Set-based symbolic computation**
  - First lower bounds and matching upper bounds for
    - fundamental objectives in graphs
    - SCC computation
    - (approximate) diameter
  - Communication Complexity is the right tool for (sub-) linear lower bounds for symbolic algorithms

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    - fundamental objectives in graphs
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Thank you for your attention!

(and see you at the poster)