Parameterized Approximation Schemes for Steiner Trees with Small Number of Steiner Vertices

Pavel Dvořák[1], Andreas Emil Feldmann[1], Dušan Knop[1,2], Tomáš Masařík[1], Tomáš Toufar[1], Pavel Veselý[1]

[1]Charles University,Prague, Czech Republic
[2]University of Bergen, Bergen, Norway

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Steiner Tree

- Input:
  - Graph $G = (V, E)$.
  - Edge weights: $w : E \to \mathbb{R}_0^+$.
  - Terminals $R \subseteq V$ (vertices in $V \setminus R$ are called Steiner vertices).
- Goal: find a Steiner tree $T \subseteq G$.
  - $R \subseteq V(T)$.
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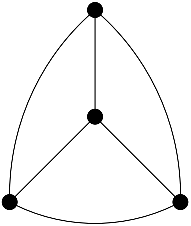

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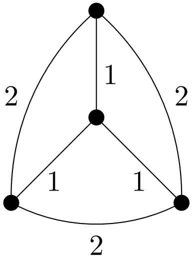

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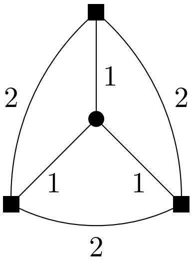

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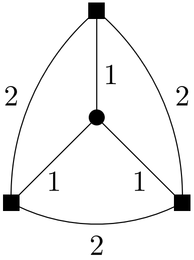

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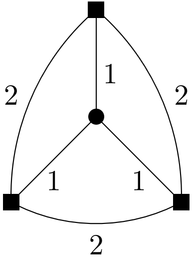

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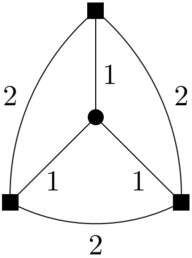

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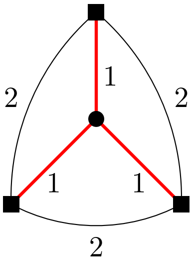

Known Results

Parameterized Complexity

- Number of terminals $|R|$.
  - FPT-algorithm [Dreyfus and Wagner '71, Mölle et al. '06].
  - $(1+\varepsilon)$-approximate polynomial size kernel [Lokshtanov et al. '16].
- Number of Steiner vertices in the optimum $|V(T) \setminus R| = p$.
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We study the parameter $p = |V(T) \setminus R|$ – good problem for **parameterized approximation**.

Our Results

Existence of

1. Efficient parameterized approximation scheme – it returns $(1+\varepsilon)$-approximation in time $f(p,\varepsilon) \cdot poly(n)$.

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	Unweighted		Weighted	
Undirected	✓	✓	✓	✓
Directed	✓	✗ *	✗ **	✗ **

* Unless NP $\subseteq$ coNP/poly.
** Unless FPT$=$ W[2].

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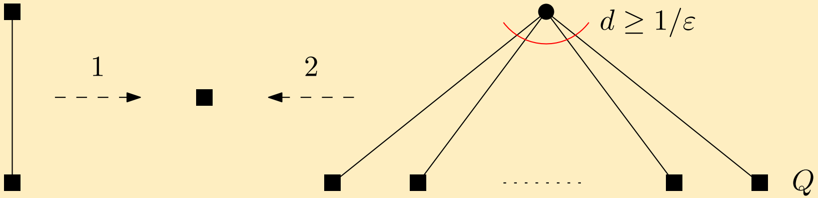

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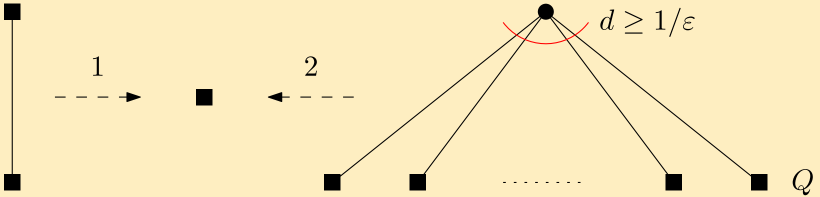

Reduction Rule 1: We can assume any such edge is in the optimal solution.

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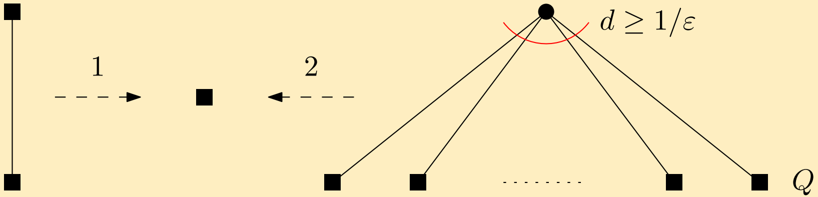

Reduction Rule 2: The optimal solution uses at least d edges to connect terminals in Q. Our solution uses at most $d + 1$ edges.

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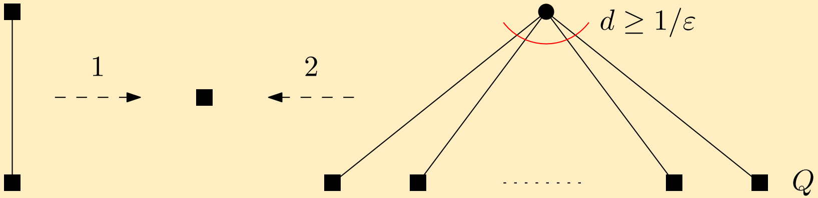

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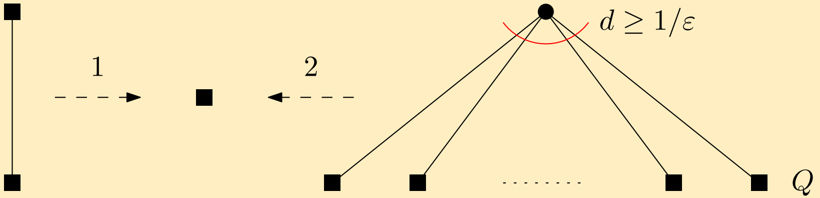

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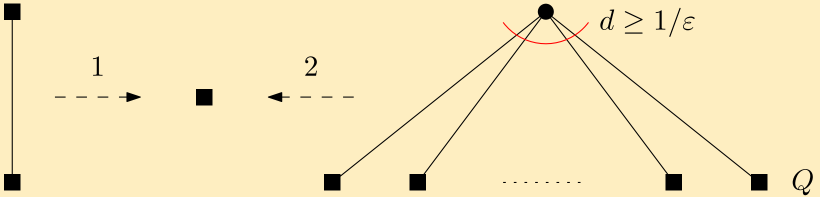

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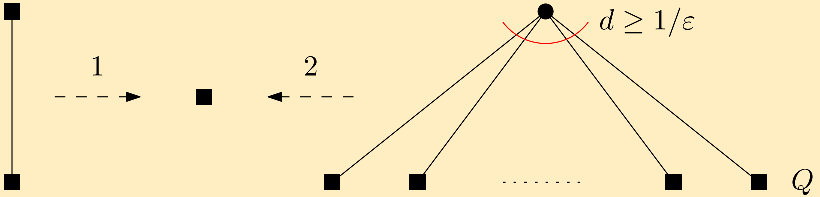

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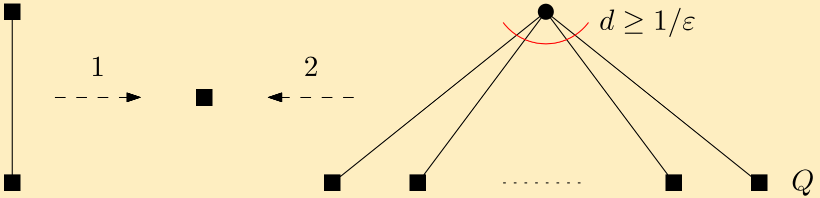

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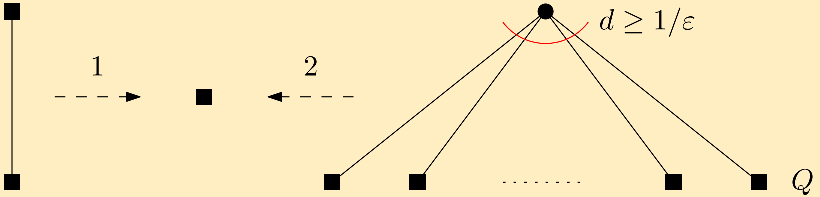

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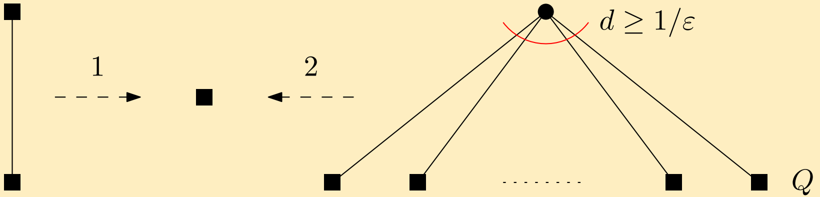

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  2. Each Steiner vertex is adjacent to at most $1/\varepsilon$ terminals.

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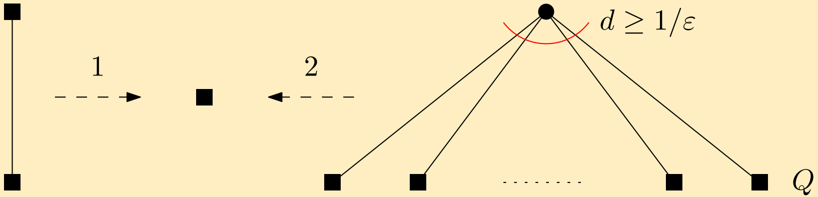

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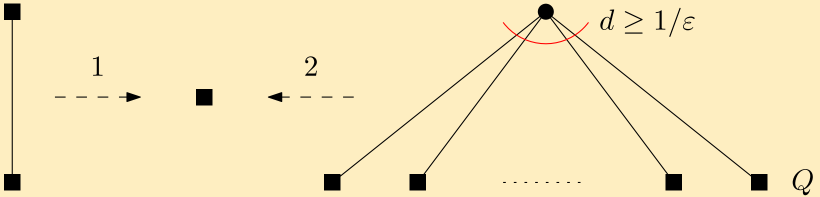

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  4. There are at most p/ε terminals.

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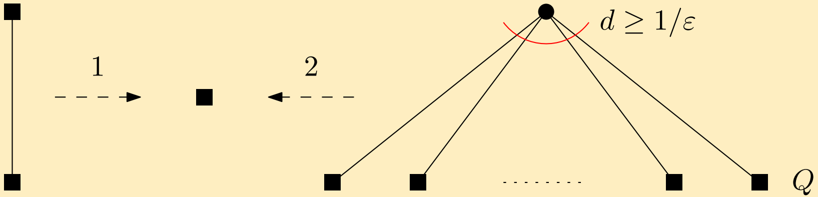

- The directed unweighted case is similar.

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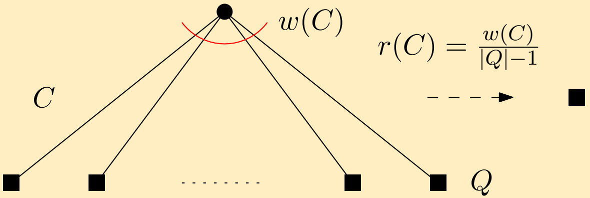
Undirected Weight Case



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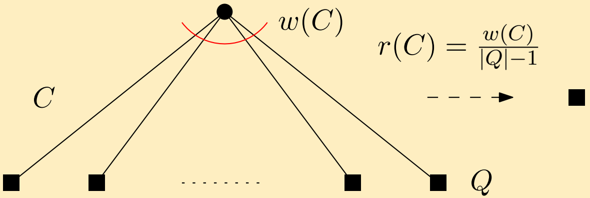

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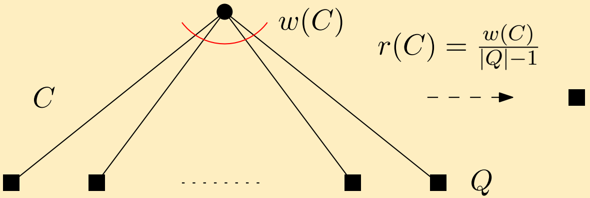

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