(Wireless) Scheduling, Graph Classes, and c-Colorable Subgraphs [1]

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Abstract

The NP-hard problem of finding maximum-weight induced c-colorable subgraphs, which is a generalization of finding maximum independent sets, naturally occurs when selecting c sets of pairwise non-conflicting jobs (modeled as graph vertices). We investigate the parameterized complexity of this problem with respect to the solution size on inductive k-independent graphs. Inductive k-independent graphs are a generalization of chordal graphs and have recently been advocated in the context of interference-avoiding wireless communication scheduling [Ásgeirsson et al, ICALP, 2017 [1] and Halldórsson and Tonoyan, STOC, 2015 [4]]. We show that the Maximum Independent Set problem is W[1]-hard even on 2-simplicial 3-minoes—a subclass of inductive 2-independent graphs. In contrast, we prove that the more general c-Colorable Subgraph problem is fixed-parameter tractable on edge-wise unions of cluster and chordal graphs, which are 2-simplicial. Aside from this, we survey other graph classes between inductive 1-independent and inductive 2-independent graphs with applications in scheduling.

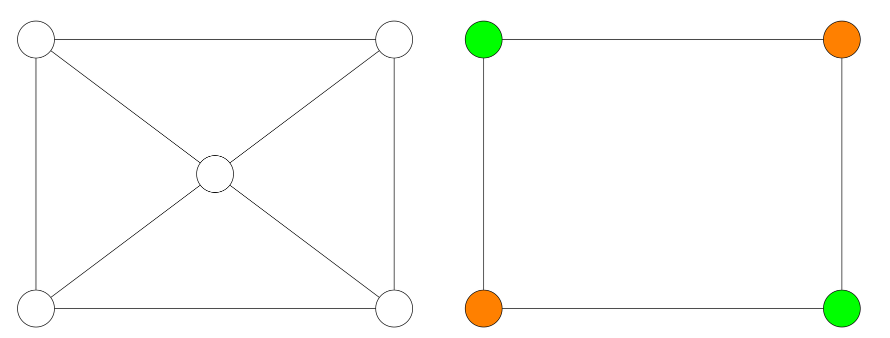
Introduction

Scheduling, routing and power control/assignment in wireless communication networks translates to c-Colorable Subgraph in inductive k-independent graphs with constant k [1].

***c*-Colorable Subgraph**
Input: A graph $G = (V, E)$ (possibly with vertex weights).
Question: Find a set $S \subseteq V$ of maximum cardinality (weight) such that $G[S]$ is c-colorable.

1-Colorable Subgraph = Maximum Independent Set

Example ($c = 2$):

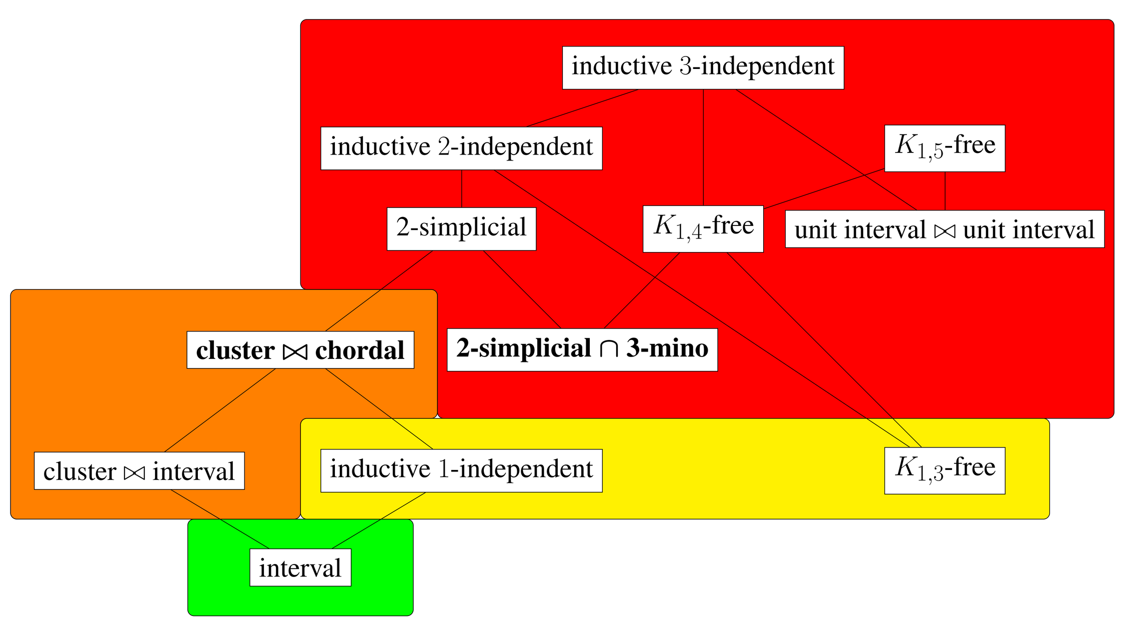


Related Work

- Maximum Independent Set parameterized by solution size is fixed-parameter tractable on strip graphs [3] (= cluster ⋈ interval graphs).
- Max-Weight Independent Set parameterized by solution size is W[1]-hard on unit 2-track interval graphs [5] (= interval ⋈ interval graphs).
- (Unweighted) c-Colorable Subgraph parameterized by solution size is fixed-parameter tractable on inductive 1-independent (= chordal) graphs [6].

Overview Of Results

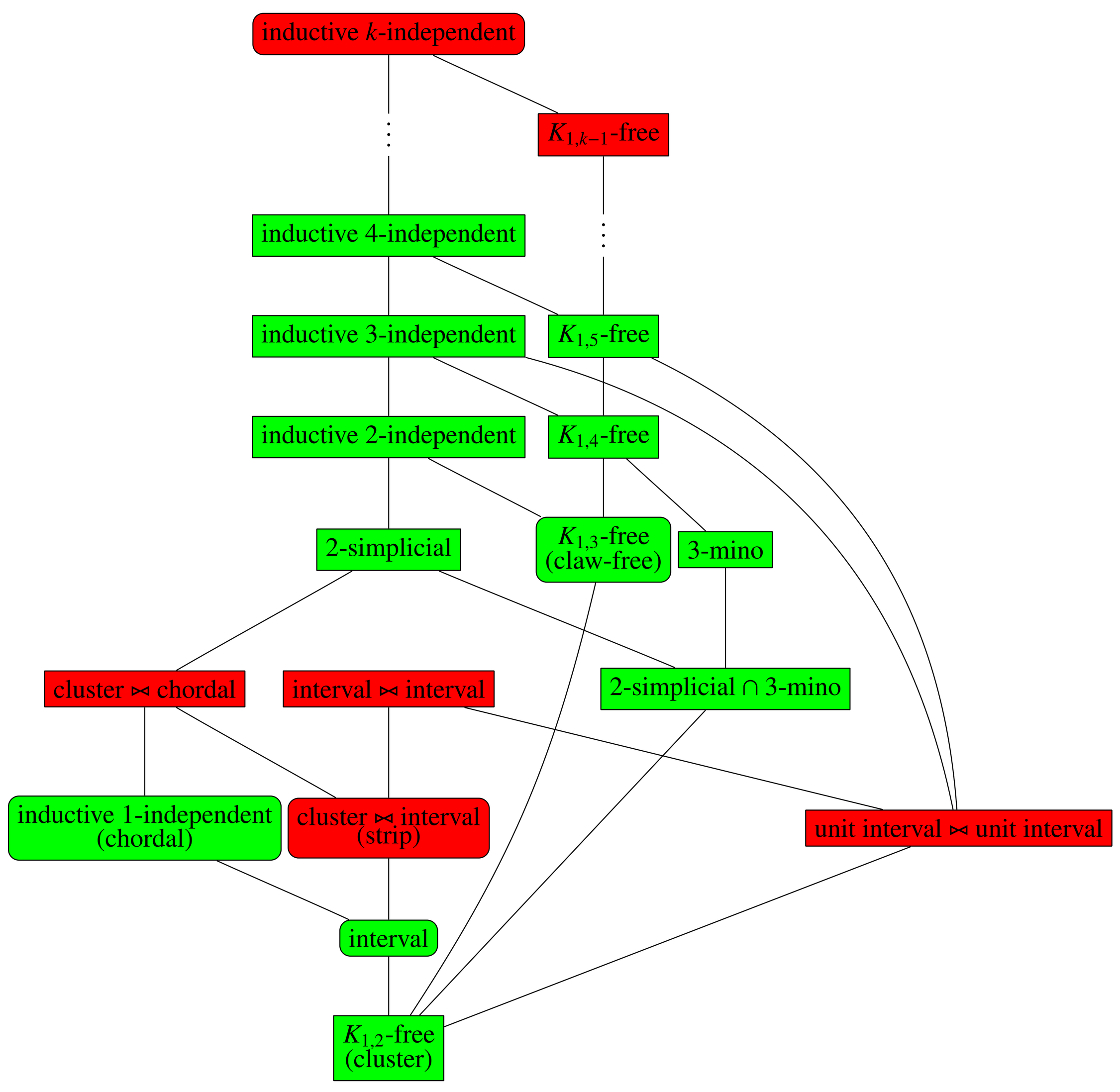
Summary of results (both new and previously known):
Legend for diagram:
green — c-Colorable Subgraph in polynomial time.
yellow — Max-Weight Independent Set in polynomial-time, c-Colorable Subgraph NP-hard but fixed-parameter tractable
orange — Maximum Independent Set NP-hard, c-Colorable Subgraph fixed-parameter tractable
red — Maximum Independent Set NP-hard and W[1]-hard
bold — new result



parameter: solution size

Graph Classes And Their Recognition

Hierarchy of graph classes and the complexity of their respective recognition problem:
Legend for diagram:
green — polynomial-time recognition
red — recognition NP-hard or W[1]-hard
edge — the lower graph class is properly contained in the upper one
rounded corners — from existing scheduling literature

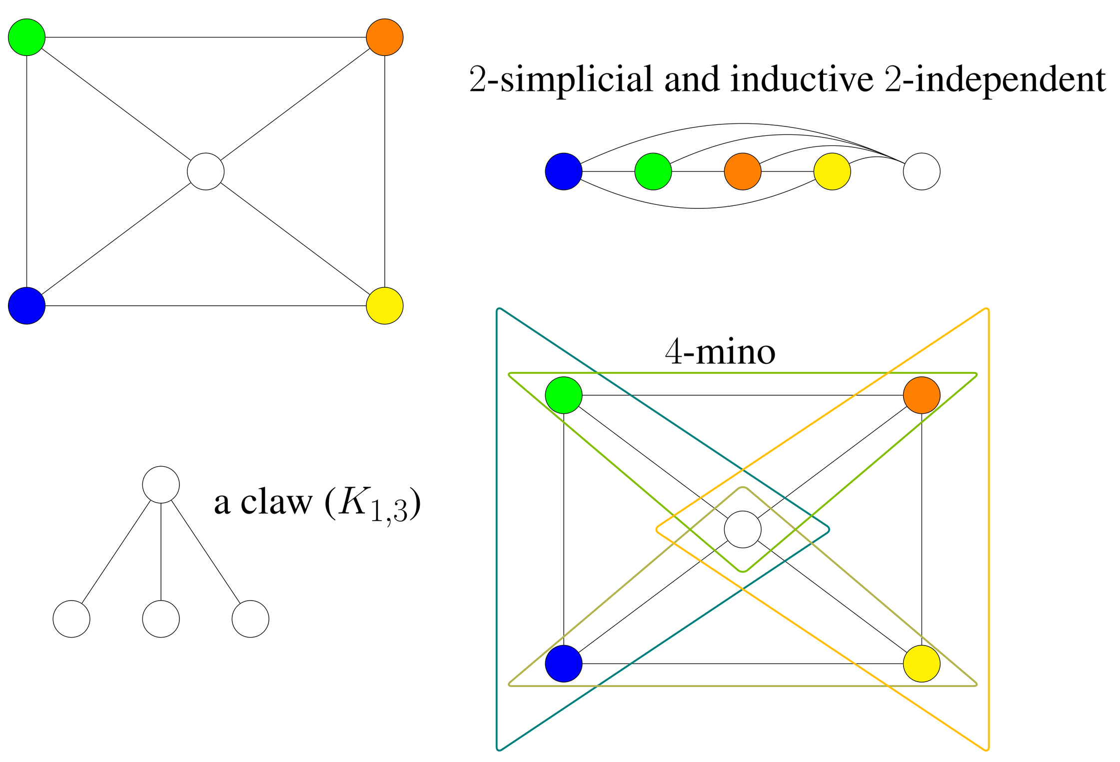


Definition ($A \bowtie B$)
For two graph classes A and B, $A \bowtie B$ is the class of graphs $G = (V, E)$ such that $E = E_1 \cup E_2$ for a graph $G_1 = (V, E_1)$ in class A and a graph $G_2 = (V, E_2)$ in class B.

graph class	definition
k-simplicial	existence of ordering such that “right-neighborhood” can be covered by k cliques
k-mino	each vertex is contained in at most k maximal cliques
$K_{1,k}$-free	does not contain a $K_{1,k}$ (a tree with one internal node and k leaves) as an induced subgraph
inductive k-independent	existence of ordering such that “right-neighborhood” only contains independent sets of size at most k

$K_{1,3}$ is usually called a claw (⇝ claw-free graphs)

Example:



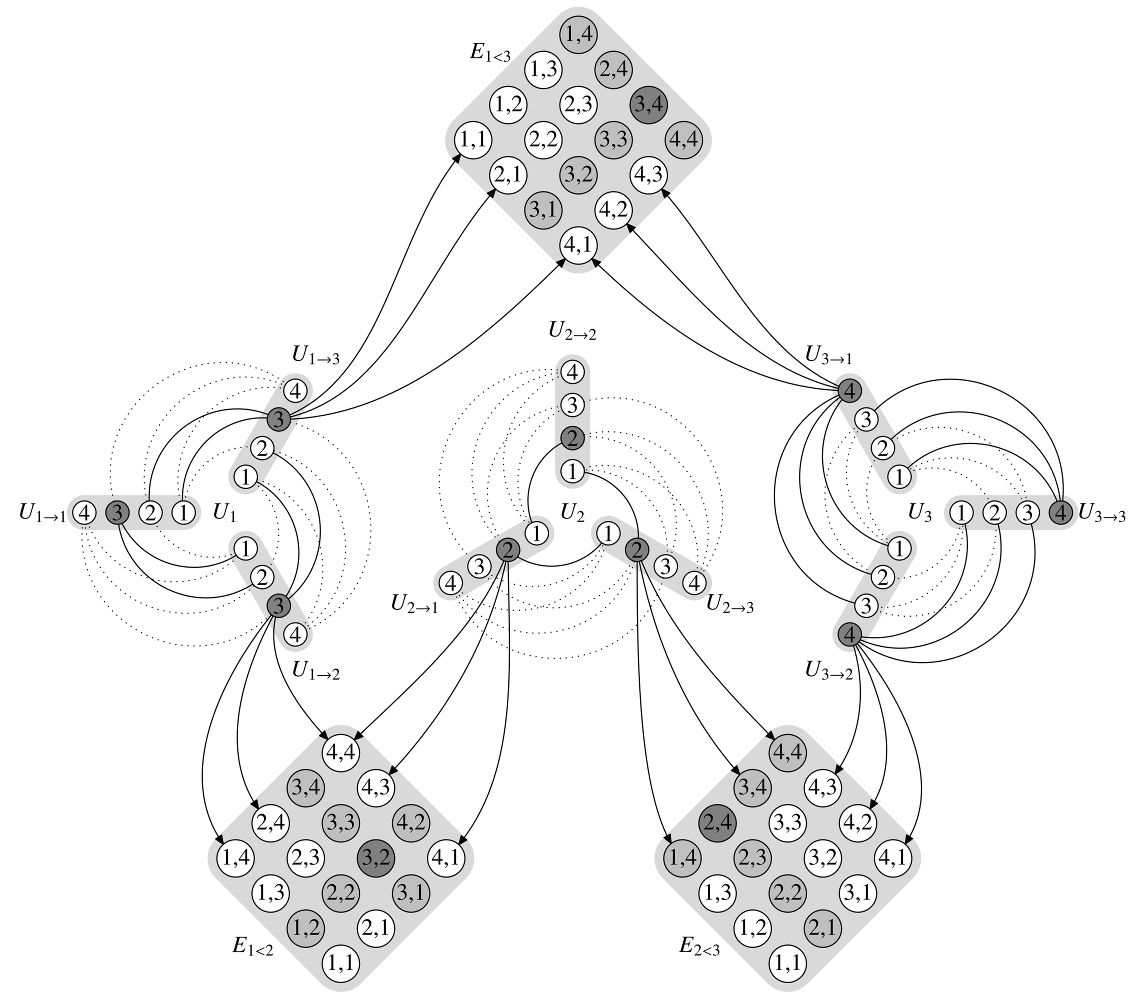
W[1]-Hardness Of Maximum Independent Set On 2-Simplicial 3-Minoes

Theorem
Maximum Independent Set parameterized by solution size ℓ is W[1]-hard on 2-simplicial 3-minoes.

Idea of Proof: Reduction from Multicolored Clique:
- color gadget for all vertices of a specific color,
- edge gadget for all edges between vertices of different colors.

Multicolored Clique
Input: A graph G whose vertex set is partitioned into independent sets $V_1 \uplus V_2 \uplus \cdots \uplus V_k$.
Question: Does G contain a clique of order k?

Illustration of the reduction from Multicolored Clique to Maximum Independent Set:
Legend for the picture: Spiral — gadget for all vertices of a color
Grid — gadget for edges between vertices of different colors
Grayshaded areas — cliques
dotted — regular edges (uncluttering the picture)
directed arc — edges between the tail vertex to all vertices in the head column, only shown for dark gray vertices



(three colors of four vertices each)

Challenge in the proof: Show that the resulting graph is 2-simplicial 3-minoe.

Idea to overcome this challenge: Always find a vertex whose neighborhood can be covered by few cliques.

Fixed-Parameter Tractability Of c-Colorable Subgraph On Cluster ⋈ Chordal

Theorem
A maximum c-colorable subgraph on at most ℓ vertices of a cluster ⋈ chordal graph can be computed in $2^{\ell+c} \cdot (c+e+3)^{\ell} \cdot \ell^{O(\log \ell)} \cdot n^2 \cdot \log^3 n$ time if the decomposition of the input graph into a cluster and a chordal graph is given. Herein, e is Euler's number.

Idea of Proof:
- Use Max-Weight Colorful Independent Set on chordal graphs as a sub-procedure and solve it using tree decomposition.
- Transfer solutions of Max-Weight Colorful Independent Set to solutions of c-Colorable Subgraph on cluster ⋈ chordal graphs using color coding two times.

Max-Weight Colorful Independent Set
Input: A graph $G = (V, E)$ with vertex weights $w\colon V \to \mathbb{N}$ and a coloring $\operatorname{col}\colon V \to \{1, 2, \ldots, c\}$ of its vertices.
Question: Find a maximum-weight independent set whose vertices have mutually distinct colors.

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[1]Long version of this work available on arXiv [2]